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An alternative block bootstrap in time series with weak dependence

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Abstract : When the observations are dependent, like in time series, Kunsch introduced the bootstrap with blocks forming by a fix number of consecutive observations. Different versions of block bootstrap has been formulated. In this paper we have proposed a bootstrap estimation with blocks formed from recalculated values of a statistic. We call it bootstrap with re-blocks. We have shown that this bootstrap works in time series strictly stationary α-mixing or m-dependent under some conditions. We have done simulations to compare the bootstrap with re-blocks with other block bootstrap methods.

Keywords : bootstrap, α-mixing, time series, weak dependence, m-dependent.

1 Introduction

The bootstrap technique [6, 7] was introduced to provide nonparametric estimates of bias and standard error. The bootstrap is biased on repeated analyses of pseudo-data created by resampling the actual

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data. However, the bootstrap requires independence, which is rarely to get in time series.

Adoptions of the basic bootstrap to time series data work by resampling sets of consecutive observations to capture the process autocorrelation structure. Kunsch [13] and Liu and Singh [19] independently introduced the moving block bootstrap (MBB), which work by randomly overlapping blocks of fixed size with replacement. Liu and Singh [19] established that MBB estimate of the sample mean is biased in finite samples but asymptotically unbiased. They also showed the consistence of MBB estimate of the variance of the sample mean.

Politis and Romano [21] proposed another scheme for stationary time series, the stationary bootstrap (SB). In the SB, the data are resampled by concatenating blocks whose starting point is chosen at random and whose length is geometrically distributed with some chosen mean. Politis and Romano established that the SB estimate of the sample mean is unbiased and the SB estimate of the variance of the sample mean is consistent.

Different versions of block bootstrap methods has been formulated by Carlstein [2], Carlstein et al. [3], Kunsch and Carlstein [14], Hall [9], Politis and Romano [20], Ekonomi and Butka [8]. Properties of block bootstrap methods have been investigated by Davison and Hall [4], Shao and Yu [24], Lahiri [15, 16, 18] and others. The important problem of choosing optimal block length has been addressed by Buhlman and Kunsch [1], Hall et al [12] and Lahiri [17].

Let us suppose that the unknown parameter μ is a parameter of the joint distribution of the strictly stationary α-mixing or m-dependent time series $X_t, t \in Z$ and $\overset{\wedge}{\theta}$ is an estimator of μ based on the observations $X_1, ..., X_N$ from this time series.

In this paper we have proposed the bootstrap with re-blocks in time series strictly stationary α-mixing or m-dependent for estimating the parameter μ. In Section 2 we have shown the idea of this bootstrap estimation. At the beginning we have formed blocks compounded by s consecutive observations from a given time series. Then we have calculated the statistic of interest $\overset{\wedge}{\theta}$ for every block to estimate the unknown parameter μ. We have considered these calculated values like the observations of a new time series and with them we have formed blocks with length b. If we have r of them, we choose randomly k blocks with the same probability $\frac{1}{r}$. We concatenate these k blocks and we have construct the bootstrap sample compounded by $m = k \times b$ observations. In Section 3 we have shown that this bootstrap method works for time series strictly stationary α-mixing or m-dependent under some conditions. In Section 4 we have shown the simulation results in various ARMA models. From the results we see that the bootstrap with re-blocks perform shorter confidence intervals than the other block bootstrap methods. This bootstrap gives similar results with other bootstrap methods regarding coverage probability, bias and root mean square error (RMSE).

2 Bootstrap with re-blocks

In many time series problems the goal is to estimate a parameter of the joint distribution. Let suppose that μ is a unknown parameter of the joint distribution of $X_t, t \in Z$. The objective is to obtain confidence intervals for μ based on some observations from time series. We will focus on estimators of μ that are in the form of an average of functions defined on the observations.

It is given the time series $X_t, t \in Z$. Suppose that we have the observations $X_1, X_2, ..., X_N$ from this time series. We create blocks compounded by s consecutive observations in the form $S_1 = \{X_1, ..., X_s\}, S_2 = \{X_2, ..., X_{s+1}\}, ..., S_{N-s+1} = \{X_{N-s+1}, ..., X_N\}$. If we denote $n = N - s + 1$, we have formed the blocks $S_1, S_2, ..., S_n$. These blocks are moving blocks.

Let suppose that $\overset{\wedge}{\theta}$ is a statistic based on the observations $X_1, X_2, ..., X_N$. We calculate $Y_1 = \overset{\wedge}{\theta}(S_1), Y_2 = \overset{\wedge}{\theta}(S_2), ..., Y_n = \overset{\wedge}{\theta}(S_n)$ and assume that $Y_1, Y_2, ..., Y_n$ are observations from a new time series $Y_t, t \in Z$. Their mean is $\overline{Y}_n = \frac{1}{n}\sum_{i=1}^{n} Y_i$. With these observations, in the same way, we can construct blocks compounded by b consecutive observations in the form $B_1 = \{Y_1, ..., Y_b\}, B_2 = \{Y_2, ..., Y_{b+1}\}, ..., B_{n-b+1} = \{Y_{n-b+1}, ..., Y_n\}$ and we denote $r = n - b + 1$. These blocks we call re-blocks.

To construct the bootstrap sample we choose randomly k blocks with probability $\frac{1}{r}$. We sign that blocks B_1^*, B_2^*, ..., B_k^*. If we concatenate these blocks consecutively, we have performed the observations $Y_1^*, Y_2^*, ..., Y_m^*$, where $m = b \times k$. This is the bootstrap sample. Their mean is

$$\overline{Y}_m^* = \frac{1}{m}\sum_{i=1}^{m} Y_i^* \tag{2.1}$$

If the parameter of interest μ is the mean of the time series $X_t, t \in Z$, then (2.1) will be the bootstrap estimator with re-blocks (BRB) for μ. To show that this bootstrap method works we will prove that the bootstrap distribution of $\sqrt{m}(\overline{Y}_m^* - \overline{Y}_n)$ approximate the distribution of $\sqrt{n}(\overline{Y}_n - \mu)$.

3 The validity of the re-blocks bootstrap

Let we do the following assumptions:

A1) $\{X_t, t \in Z\}$ is strictly stationary and α-mixing time series,

A2) $E\left|Y_1\right|^{2p+\delta} < c$, where p is integer, $p > 2, 0 < \delta \leq 2$ and $c > 0$,

A3) $EY_1 = \mu + o(\frac{1}{\sqrt{n}})$,where μ is a parameter of the joint distribution of $X_t, t \in Z$,

A4) $\sqrt{n}(\overline{Y}_n - E\overline{Y}_n) \xrightarrow{d} N(0, \sigma_\infty^2)$, where $0 < \sigma_\infty^2 < \infty$.

A5) $b = o(n)$ and if $n \longrightarrow \infty$, then $m \longrightarrow \infty$ reciprocally,

A6) $\sum_{k=1}^{\infty} k^{p-1}(\alpha_X(k))^{\frac{\delta}{2p+\delta}}$, for p integer, $p > 2$ and $0 < \delta \leq 2$.

Assumption A3 shows that the asymptotic order of the bias of $\overline{Y}_n$ is smaller than the asymptotic order of its standard deviations. We note that the assumptions A3 and A4 allows us to consider confidence intervals for $E\overline{Y}_n$ as confidence intervals for μ asymptotically, since by Slutsky's theorem, we have

$$\sqrt{n}(\overline{Y}_n - \mu) \xrightarrow{d} N(0, \sigma_\infty^2).$$

This asymptotic normal distribution can be used to yield confidence intervals for μ. However, the variance of σ_∞^2 must be estimated, but in many cases it is not feasible. In addition, a different estimate of the sampling distribution of $\sqrt{n}(\overline{Y}_n - \mu)$ might gives a better approximation, thus giving confidence intervals that are more accurate. It is the role that the bootstrap is usually called to play.

In the following treatments all the limits are calculated when $N \to \infty$ and $m \to \infty$. In this case, form the relevant relations, we will understand that $n \to \infty$ and $r \to \infty$ and conversely.

Now we see some lemma and theorems that will help us to prove the validity of the proposed bootstrap.

Lemma 3.1. *If the conditions A1-A6 are true, then*

$$\frac{1}{r}\sum_{i=1}^{r} Z_i \xrightarrow{p} EZ_1, \frac{1}{r}\sum_{i=1}^{r} |Z_i| \xrightarrow{p} E\left|Z_1\right|, \frac{1}{r}\sum_{i=1}^{r} |Z_i|^3 \xrightarrow{p} E\left|Z_1\right|^3,$$

where $Z_i = \frac{1}{\sqrt{b}}\sum\limits_{j=i}^{i+b-1} Y_j, i = 1, 2, ..., r.$

Proof. Since the time series $X_t, t \in Z$ is strictly stationary and α-mixing time series, also the time series $Y_t, t \in Z$ is strictly stationary and α-mixing with $\alpha_Y(t) \leq \alpha_X(t - s + 1)$. We have $\frac{1}{r}\sum\limits_{i=1}^{r} Z_i - EZ_1 = \frac{1}{r}\sum\limits_{i=1}^{r}(Z_i - EZ_i)$. Then, based on some moment inequalities for mixing sequences [22, 23, 25, 26], we have

$$var(\frac{1}{r}\sum_{i=1}^{r}(Z_i - EZ_i)) =$$

$$= \frac{1}{r}var(Z_1 - EZ_1) + \frac{2}{r^2}\sum_{i=1}^{r-1}(r-i)cov(Z_1 - EZ_1, Z_{i+1} - EZ_{i+1}) \leq$$

$$\leq \frac{10}{r}(E\left|Z_1 - EZ_1\right|^p)^{\frac{2}{p}}(\frac{1}{2})^{\frac{p-2}{p}} + \frac{20}{r^2}\sum_{i=1}^{r-1}(r-i)(E\left|Z_1 - EZ_1\right|^p)^{\frac{2}{p}}(\alpha_Y(i))^{\frac{p-2}{p}} \leq$$

$$\leq \frac{10}{r}(K(E\left|Y_1\right|^{2p+\delta})^{\frac{2}{2p+\delta}}(\frac{1}{2})^{\frac{p-2}{p}} + \frac{20}{r^2}\sum_{i=1}^{r-1}(r-i)(KE\left|Y_1\right|^{2p+\delta})^{\frac{2}{2p+\delta}}(\alpha_Y(i))^{\frac{p-2}{p}} =$$

$$= O(\frac{1}{r} + \frac{20}{r^2}\sum_{i=1}^{r-1}(r-i)(\alpha_Y(i))^{\frac{p-2}{p}})$$

From A6, the series $\sum\limits_{k=1}^{\infty} k\alpha_X(k)^{\frac{p-2}{p}}$ and $\sum\limits_{k=1}^{\infty} \alpha_X(k)^{\frac{p-2}{p}}$ are convergent.

Thus we have $var(\frac{1}{r}\sum\limits_{i=1}^{r}(Z_i - EZ_i)) \xrightarrow{p} 0$. So, by the Chebyshev theorem we have $\frac{1}{r}\sum\limits_{i=1}^{r} Z_i \xrightarrow{p} EZ_1$. Then, from the properties of the convergence in probability we have $\frac{1}{r}\sum\limits_{i=1}^{r} |Z_i| \xrightarrow{p} E\left|Z_1\right|$ and $\frac{1}{r}\sum\limits_{i=1}^{r} |Z_i|^3 \xrightarrow{p} E\left|Z_1\right|^3$. □

Lemma 3.2. *If the conditions A1-A6 are true, then* $\frac{1}{r}\sum_{i=1}^{r}(Z_i-\sqrt{b}\overline{Y}_n)^2 \xrightarrow{p} \sigma_\infty^2$.

Proof. We have

$$\frac{1}{r}\sum_{i=1}^{r}(Z_i-\sqrt{b}\overline{Y}_n)^2 = \frac{1}{r}\sum_{i=1}^{r}(Z_i-EZ_i-\sqrt{b}(\overline{Y}_n-\frac{1}{\sqrt{b}}EZ_i))^2 =$$

$$= A_n - 2C_n + D_n,$$

where

$$A_n = \frac{1}{r}\sum_{i=1}^{r}(Z_i-EZ_i)^2,$$

$$C_n = \frac{1}{r}\sum_{i=1}^{r}\sqrt{b}(Z_i-EZ_i)(\overline{Y}_n-\frac{1}{\sqrt{b}}EZ_i),$$

and

$$D_n = \frac{1}{r}\sum_{i=1}^{r}b(\overline{Y}_n-\frac{1}{\sqrt{b}}EZ_i)^2.$$

Let we analyze these terms

1) In similar way with Lemma 3.1, we have

$$varA_n \le \frac{10}{r}(E\left|Z_1-EZ_1\right|^{2p})^{\frac{2}{p}}(\frac{1}{2})^{\frac{p-2}{p}}+$$

$$+\frac{20}{r^2}\sum_{i=1}^{r-1}(r-i)(E\left|Z_1-EZ_1\right|^{2p})^{\frac{2}{p}}(\alpha_Y(i))^{\frac{p-2}{p}}$$

or

$$varA_n \le \frac{10}{r}K(E\left|Y_1\right|^{2p+\delta})^{\frac{4}{2p+\delta}}+$$

$$+\frac{20}{r^2}\sum_{i=1}^{r-1}(r-i)K(E\left|Y_1\right|^{2p+\delta})^{\frac{4}{2p+\delta}}(\alpha_Y(i))^{\frac{p-2}{p}} =$$

$$= O(\frac{1}{r}+\frac{20}{r^2}\sum_{i=1}^{r-1}(r-i)(\alpha_Y(i))^{\frac{p-2}{p}}).$$

We see that $varA_n \xrightarrow{p} 0$. Now, from the condition A3, we have

$$EA_n = varZ_1 = var(\frac{1}{\sqrt{b}}\sum_{j=1}^{b}Y_j) = var(\sqrt{b}(\frac{1}{b}\sum_{j=1}^{b}Y_j)) \xrightarrow{p} \sigma_\infty^2.$$

Based on the Chebyshev theorem we see that $A_n \xrightarrow{p} \sigma_\infty^2$.

2) Let we consider the term C_n.

$$C_n = \sqrt{b}(\overline{Y}_n-\frac{1}{\sqrt{b}}EZ_1)\frac{1}{r}\sum_{i=1}^{r}\sqrt{b}(Z_i-EZ_1),$$

In Lemma 3.1 we have shown that $var(\frac{1}{r}\sum_{i=1}^{r}(Z_i - EZ_i)) \xrightarrow{p} 0$ and $E(\frac{1}{r}\sum_{i=1}^{r}(Z_i - EZ_i)) \xrightarrow{p} 0$. By the conditions A2, A3 and A4 we have $\sqrt{b}(\overline{Y}_n - \frac{1}{\sqrt{b}}EZ_1) = o_p(1)$. Based on Chebyshev theorem we have $C_n \xrightarrow{p} 0$.

3) We have

$$D_n = \frac{1}{r}\sum_{i=1}^{r} b(\overline{Y}_n - \frac{1}{\sqrt{b}}EZ_i)^2 =$$

$$= \frac{1}{r}\sum_{i=1}^{r} b(\overline{Y}_n - \mu)^2 + b(\mu - \frac{1}{\sqrt{b}}EZ_i)^2 + 2b(\overline{Y}_n - \mu)(\mu - \frac{1}{\sqrt{b}}EZ_i).$$

Let analyze separately the three terms:

a) From the conditions A2 and A3 we have

$$\sqrt{b}(\overline{Y}_n - \mu) = \sqrt{b}(\overline{Y}_n - E\overline{Y}_n) + \sqrt{b}E(\overline{Y}_n - \mu) =$$

$$= \frac{\sqrt{b}}{\sqrt{n}}\sqrt{n}(\overline{Y}_n - E\overline{Y}_n) + \sqrt{b}o(n^{-\frac{1}{2}}) \xrightarrow{p} 0.$$

Then

$$b(\overline{Y}_n - \mu)^2 = \frac{b}{n}(\sqrt{n}(\overline{Y}_n - \mu))^2 \xrightarrow{p} 0.$$

b)

$$b(\overline{Y}_n - \mu)(\mu - \frac{1}{\sqrt{b}}EZ_i) = b(\overline{Y}_n - \mu)(\mu - EY_1) = o_p(bn^{-1}) \xrightarrow{p} 0.$$

c)

$$b(\mu - \frac{1}{\sqrt{b}}EZ_i)^2 = o(bn^{-1}) \xrightarrow{p} 0.$$

Then $D_N \xrightarrow{p} 0$. Joining the convergence results of A_n, C_n, D_n we see the truth of lemma. □

Theorem 3.1. *If the conditions A1, A2 and A5 are fulfilled, then* $E^*\overline{Y}^*_m = \overline{Y}_n + o_p(1)$.

Proof. We have

$$E^*\overline{Y}^*_m = E^*\frac{1}{k}\sum_{i=1}^{k}\frac{1}{b}\sum_{j=(i-1)b+1}^{ib} Y^*_j = E^*\frac{1}{k}\sum_{i=1}^{k}\frac{1}{\sqrt{b}}Z^*_i = E^*\frac{1}{\sqrt{b}}Z^*_1,$$

where $Z^*_i = \frac{1}{\sqrt{b}}\sum_{j=(i-1)b+1}^{ib} Y^*_j$.

From the other hand

$$E^*Z^*_1 = \frac{1}{r}\sum_{i=1}^{r} Z_i = \frac{1}{r}\sum_{i=1}^{r}\frac{1}{\sqrt{b}}\sum_{j=i}^{i+b-1} Y_j = \frac{1}{\sqrt{b}}(b\overline{Y}_n - \frac{1}{r}\sum_{i=1}^{b-1}(b-i)(Y_i + Y_{n-i+1})).$$

Now

$$E^*\overline{Y}^*_m = \frac{1}{b}(b\overline{Y}_n - \frac{1}{r}\sum_{i=1}^{b-1}(b-i)(Y_i + Y_{n-i+1}).$$

From the condition A2 and the Chebychev theorem we see that $Y_i, i = 1, 2, ..., n$ are bounded in probability. To finish the proof of the theorem we show that

$$\left|\frac{1}{rb}\sum_{i=1}^{b-1} i(Y_i + Y_{n-i+1})\right| \leq \frac{1}{rb}\sum_{i=1}^{b-1}(b-i)(|Y_i| + |Y_{n-i+1}|) \leq \frac{2c_1(b-1)}{r} = o_p(1).$$

□

Theorem 3.2. *If the conditions A1-A6 are true, then* $m \cdot var^*(\overline{Y}^*_m) \xrightarrow{p} \sigma^2_\infty$.

Proof. Z^*_i for $i = 1, 2, ..., k$ are independent. We see that

$$m \cdot var^*(\overline{Y}^*_m) = m \cdot var^*(\frac{1}{m}\sum_{i=1}^{m} Y^*_i) = m \cdot var^*(\sum_{i=1}^{k}\frac{1}{k}\sum_{j=(i-1)b+1}^{ib} Y^*_i) =$$

$$= m \cdot var^*(\frac{1}{k}\sum_{i=1}^{k}\frac{1}{\sqrt{b}}Z^*_i) = var^*(Z^*_1).$$

But

$$var^*(Z^*_1) = \frac{1}{r}\sum_{i=1}^{r}(Z_i - E^*Z^*_i)^2 = \frac{1}{r}\sum_{i=1}^{r}(Z_i - \sqrt{b}\overline{Y}_n + o_p(1))^2.$$

This expression has the same limit with the expression $\frac{1}{r}\sum_{i=1}^{r}(Z_i - \sqrt{b}\overline{Y}_n)^2$ that converges in probability to σ^2_∞ based on the result of Lemma 3.2. Now it is clear the truth of the theorem. □

Theorem 3.3. *If the conditions A1-A6 are fulfilled, then*
$\sup_x \left|p^*\{\frac{\overline{Y}^*_m - E^*\overline{Y}^*_m}{\sqrt{var^*_m\overline{Y}^*}} \leq x\} - \Phi(x)\right| \xrightarrow{p} 0.$

Proof. We have

$$\overline{Y}^*_m = \frac{1}{k}\sum_{i=1}^{k}\frac{1}{b}\sum_{j=(i-1)b+1}^{ib} Y^*_j = \frac{1}{k\sqrt{b}}\sum_{i=1}^{k} Z^*_i.$$

From this we have

$$E^*\overline{Y}^*_m = \frac{1}{k\sqrt{b}}\sum_{i=1}^{k} E^*Z^*_i$$

and

$$\sqrt{var^*\overline{Y}^*_m} = \frac{1}{\sqrt{b}}\sqrt{\frac{1}{k}\sum_{i=1}^{k} var^*Z^*_i}.$$

So

$$\frac{\overline{Y}^*_m - E^*\overline{Y}^*_m}{\sqrt{var^*\overline{Y}^*_m}} = \frac{\frac{1}{k}\sum_{i=1}^{k} Z^*_i - \frac{1}{k}\sum_{i=1}^{k} EZ^*_i}{\sqrt{\frac{1}{k}\sum_{i=1}^{k} var^*Z^*_i}}.$$

We can apply Berry-Esseen theorem and we have

$$\sup_x \left| p^*\{\frac{\overline{Y}^*_m - E^*\overline{Y}^*_m}{\sqrt{var^*\overline{Y}^*_m}} \le x\} - \Phi(x) \right| = \sup_x \left| p^*\{\frac{\frac{1}{k}\sum\limits_{i=1}^{k} Z_i^* - \frac{1}{k}\sum\limits_{i=1}^{k} EZ_i^*}{\sqrt{\frac{1}{k}\sum\limits_{i=1}^{k} var^* Z_i^*}} \le x\} - \Phi(x) \right| \le$$

$$\le \frac{E^*\left|Z_1^* - EZ_1^*\right|^3}{\sqrt{var^* Z_1^*}}.$$

But

$$E^*\left|Z_1^* - EZ_1^*\right|^3 = \frac{1}{r}\sum_{i=1}^{r}\left| Z_i - \frac{1}{r}\sum_{j=1}^{r} Z_j \right|^3.$$

We can apply the Minkowski inequality and we have

$$\left(\sum_{i=1}^{r}\left| Z_i - \frac{1}{r}\sum_{j=1}^{r} Z_j \right|^3\right)^{\frac{1}{3}} \le \left(\sum_{i=1}^{r}\left|Z_i\right|^3\right)^{\frac{1}{3}} + \left(\sum_{i=1}^{r}\left|\frac{1}{r}\sum_{j=1}^{r} Z_j\right|^3\right)^{\frac{1}{3}} =$$

$$= \left(\sum_{i=1}^{r}\left|Z_i\right|^3\right)^{\frac{1}{3}} + \frac{1}{r^{\frac{2}{3}}}\left|\sum_{j=1}^{r} Z_j\right|.$$

Then

$$E^*\left|Z_1^* - EZ_1^*\right|^3 \le \frac{1}{r}\left(\left(\sum_{i=1}^{r}\left|Z_i\right|^3\right)^{\frac{1}{3}} + \frac{1}{r^{\frac{2}{3}}}\left|\sum_{j=1}^{r} Z_j\right|\right)^3 =$$

$$= \left(\left(\frac{1}{r}\sum_{i=1}^{r}\left|Z_i\right|^3\right)^{\frac{1}{3}} + \left|\frac{1}{r}\sum_{j=1}^{r} Z_j\right|\right)^3.$$

From Lemma 3.1 we take

$$\left(\left(\frac{1}{r}\sum_{i=1}^{r}\left|Z_i\right|^3\right)^{\frac{1}{3}} + \left|\frac{1}{r}\sum_{j=1}^{r} Z_j\right|\right)^3 \xrightarrow{p} \left(\left(E\left|Z_1\right|^3\right)^{\frac{1}{3}} + \left|EZ_1\right|\right)^3.$$

From this convergence we conclude that $E^*\left|Z_1^* - EZ_1^*\right|^3$ is bounded in probability. It is clear enough that the proof is complete. □

Now we prove the main result that shows the validity of the proposed bootstrap. This result allows us to justify the construction of confidence intervals for μ based on the distribution of the bootstrap with re-blocks.

Theorem 3.4. *If the conditions A1-A6 are fulfilled, then*

$$\sup_x \left| p^*\{\frac{\overline{Y}_m^* - E^*\overline{Y}_m^*}{\sqrt{var^*\overline{Y}_m^*}} \leq x\} - p\{\sqrt{n}(\overline{Y}_n - \mu) \leq x\} \right| \xrightarrow{p} 0.$$

Proof. From assumptions A3 and A4 we have

$$\sup_x \left| p\{\sqrt{n}(\overline{Y}_n - \mu) \leq x\} - \Phi(\frac{x}{\sigma_\infty}) \right| \xrightarrow{p} 0. \tag{3.1}$$

From Theorem 3.1 we have

$$p^*\{\sqrt{m}(\overline{Y}_m^* - \overline{Y}_n) \leq x\} = p^*\{\frac{\overline{Y}_m^* - E^*\overline{Y}_m^*}{\sqrt{var^*\overline{Y}_m^*}} \leq o_p(1) + \frac{x}{\sqrt{m \cdot var^*\overline{Y}_m^*}}\},$$

and from Theorem 3.2 and 3.3 we have

$$\underset{x}{sup} \left| p^*\{\sqrt{m}(\overline{Y}_m^* - \overline{Y}_n) \leq x\} - \Phi(\frac{x}{\sigma_\infty}) \right| \xrightarrow{p} 0. \tag{3.2}$$

Now, from the inequality

$$\sup_x \left| p^*\{\frac{\overline{Y}_m^* - E^*\overline{Y}_m^*}{\sqrt{var^*\overline{Y}_m^*}} \leq x\} - p\{\sqrt{n}(\overline{Y}_n - \mu) \leq x\} \right| \leq$$

$$\leq \sup_x \left| p^*\{\sqrt{m}(\overline{Y}_m^* - \overline{Y}_n) \leq x\} - \Phi(\frac{x}{\sigma_\infty}) \right| +$$

$$+\sup_x \left| p\{\sqrt{n}(\overline{Y}_n - \mu) \leq x\} - \Phi(\frac{x}{\sigma_\infty}) \right|$$

and the relations (3.1) and (3.2) the truth of this theorem is evident. □

We showed with above theorem that the proposed bootstrap estimation works in the case of time series that are strictly stationary and α-mixing. Now let suppose that the time series $X_t, t \in Z$ is strictly stationary and m-dependent. In this case $\alpha_X(k) = 0$ for $k > m$. From this we take that the time series $Y_t, t \in Z$ is $m - s + 1$-dependent. If we analyze the assumption A6, we see that $\sum_{k=1}^{\infty} k^{p-1}(\alpha_X(k))^{\frac{\delta}{2p+\delta}}$ is a finite sum. So, the proposed above bootstrap estimator works in the case of time series strictly stationary and m-dependent.

4 Simulation study

This section investigates the performance of the bootstrap with re-blocks for the confidence intervals estimation, when the parameter of interest is the time series mean of an $ARMA$ process. We consider $AR(1)$, $AR(2)$, $MA(1)$, $MA(2)$ and $ARMA(1,1)$ processes. In order to get a comparison to other bootstrap methods were used a wide range of coefficient values for the $AR(1)$ and $MA(1)$ processes. Coefficient values were chosen such that satisfy the stationarity and invertibility conditions.

For each model we generated time series of length n from 100 to 1000 with increments of 100. We implemented the moving block bootstrap (MBB) [13], the non-overlapping block bootstrap (NBB) [2], stationary bootstrap (SP) [21] and bootstrap with cycling blocks (BCB) [8] in order to estimate and to construct confidence intervals for the mean of time series. We constructed the percentile bootstrap (PB) and the bias corrected bootstrap (CB) intervals for the mean [5, 10]. The nominal level of the intervals was chosen to be 0.95. We used $Q = 1000$ bootstrap replications [11]. The block length was chosen at order $O(n^{\frac{1}{3}})$ for all methods [17]. We used 500 Monte Carlo replications for each simulation case to calculate the bias and the RMSE for each bootstrap point estimator and the average interval length and the empirical coverage probability in percentage for each type of intervals. R software was used. Some of the simulation results are shown in following tables.

From Tables 1,2 and 3 we see that BRB give shorter confidence intervals for time series mean μ. The other simulations results are similar between them.

Size (n)	coeff.	BCB	MBB	SB	BRB
100	0.1	0.4159	0.4098	0.4067	0.3397
	0.4	0.5776	0.5506	0.5533	0.4889
	0.7	1.0587	0.8991	0.9425	0.8215
	0.85	1.8110	1.3361	1.4258	1.2569
300	0.1	0.2453	0.2488	0.2406	0.2159
	0.4	0.3558	0.3403	0.3414	0.3118
	0.7	0.6864	0.5664	0.6128	0.5497
	0.85	1.2977	0.8855	1.0105	0.8840
500	0.1	0.1924	0.1911	0.1885	0.1725
	0.4	0.2858	0.2697	0.2702	0.2521
	0.7	0.5497	0.4592	0.4856	0.4503
	0.85	1.0680	0.7268	0.8363	0.7352
700	0.1	0.1633	0.1914	0.1617	0.1486
	0.4	0.2417	0.2713	0.2304	0.2171
	0.7	0.4734	0.4019	0.4227	0.3937
	0.85	0.9097	0.7624	0.7333	0.6569
1000	0.1	0.1365	0.1363	0.1346	0.1249
	0.4	0.2042	0.1954	0.1946	0.1850
	0.7	0.4038	0.3512	0.3590	0.3401
	0.85	0.7824	0.5812	0.6354	0.5711

Table 1. The confidence interval length for 95% PB intervals in $AR(1)$ model for various values of coefficient φ.

Size (n)	coeff.	BCB	MBB	SB	BRB
100	0.1	0.4043	0.4098	0.4004	0.3363
	0.4	0.5033	0.5098	0.4973	0.4265
	0.7	0.6026	0.6045	0.5898	0.5031
	0.85	0.6554	0.6597	0.6406	0.5592
300	0.1	0.2408	0.2422	0.2401	0.2151
	0.4	0.3052	0.3021	0.3003	0.2717
	0.7	0.3686	0.3648	0.3606	0.3304
	0.85	0.4074	0.3957	0.3945	0.3579
500	0.1	0.1895	0.1881	0.1869	0.1697
	0.4	0.2417	0.2380	0.2335	0.2137
	0.7	0.2916	0.2852	0.2852	0.2636
	0.85	0.31.84	0.3086	0.3066	0.2834
700	0.1	0.1600	0.1601	0.1588	0.1470
	0.4	0.2038	0.2013	0.1998	0.1850
	0.7	0.2465	0.2427	0.2420	0.2244
	0.85	0.2674	0.2639	0.2636	0.2458
1000	0.1	0.1362	0.1346	0.1344	0.1244
	0.4	0.1733	0.1954	0.1676	0.1575
	0.7	0.2082	0.2051	0.2029	0.1918
	0.85	0.2253	0.2227	0.2218	0.2080

Table 2. The interval length for 95% interval length in $MA(1)$ model with various values of coefficient θ.

Size (n)	$ARMA$ model	BCB	MBB	SB	BRB
100	$AR(2)$	0.8557	0.7903	0.7766	0.7013
	$MA(2)$	0.8176	0.7818	0.7615	0.6783
	$ARMA(1,1)$	1.1614	1.0566	1.0649	0.9361
300	$AR(2)$	0.5419	0.4906	0.4915	0.4597
	$MA(2)$	0.4905	0.4783	0.4719	0.4365
	$ARMA(1,1)$	0.7613	0.6805	0.6954	0.6385
500	$AR(2)$	0.4248	0.3921	0.3898	0.3646
	$MA(2)$	0.3881	0.3784	0.3781	0.3530
	$ARMA(1,1)$	0.6181	0.5406	0.5585	0.5148
700	$AR(2)$	0.3599	0.3361	0.3357	0.3160
	$MA(2)$	0.3351	0.3220	0.3223	0.3029
	$ARMA(1,1)$	0.5277	0.4690	0.4785	0.4510
1000	$AR(2)$	0.3062	0.2841	0.2840	0.2711
	$MA(2)$	0.2792	0.2711	0.2696	0.2565
	$ARMA(1,1)$	0.4461	0.4024	0.4069	0.3852

Table 3. The average interval length for 95% PB intervals for the mean with MBCB. The case: $AR(2)$ model with $\varphi_1 = 0.7, \varphi_2 = -0.1, MA(2)$ model with $\theta_1 = 0.8, \theta_2 = 0.5, ARMA(1,1)$ with $\varphi = 0.65$ and $\theta = 0.3$.

5 Conclusions

From our theoretical studies and simulation results we show that bootstrap with re-blocks (BRB) is a very good alternative for estimations in time series when the parameter of interests is a parameter of the joint distribution of a strictly stationary $\alpha-$mixing or $m-$dependent time series. We have shown that the bootstrap distribution of BRB approximates the distribution of $\sqrt{n}(\overline{X}_n - \mu)$.

So we can construct successfully confidence intervals for the unknown time series parameters estimated by a statistic $\overset{\wedge}{\theta}$ that satisfy some conditions. We have seen this fact from the simulation results. From the simulation results we see that BRB perform shorter confidence intervals than MBB, NBB, SP and BCB. The result for empirical coverage probability, bias and RMSE are similar with the other block bootstrap methods.

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On Edge Product Cordial Labeling of Some Product Related Graphs

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Abstract : An edge product cordial labeling is a variant of product cordial labeling. We have explored this concept in the context of different graph products.

Keywords : Graph Product, Product Cordial Labeling, Edge Product Cordial Labeling.

1 Introduction

We begin with simple, finite and undirected graph $G = (V(G), E(G))$ with order p and size q. For any graph theoretic notations and terminology, we follow West [9].

A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

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For a comprehensive bibliography of papers on the concept of graph labeling, readers are advised to refer Gallian [2]. The present paper is focused on edge product cordial labeling of graphs.

In 1987, Cahit [1] introduced cordial labeling as a weaker version of graceful labeling and harmonious labeling. In 2004, Sundaram *et al.* [4] have introduced the product cordial labeling in which the absolute difference of vertex labels in cordial labeling is replaced by the product of the vertex labels.

Vaidya and Barasara [5] have introduced the edge analogue of product cordial labeling and named it as edge product cordial labeling which is defined as follows.

For a graph $G = (V(G), E(G))$, an edge labeling function $f : E(G) \to \{0, 1\}$ induces a vertex labeling function $f^* : V(G) \to \{0, 1\}$ defined as $f^*(v) = \prod f(e_i)$ for $\{e_i \in E(G)/e_i$ is incident to $v_i\}$. Then f is called *edge product cordial labeling* of graph G if absolute difference of number of vertices with label 1 and label 0 is atmost 1 and absolute difference of number of edges with label 1 and label 0 is atmost 1. A graph G is called *edge product cordial* if it admits an edge product cordial labeling.

In [5, 6, 8], Vaidya and Barasara have proved several results on this newly defined concept while the edge product cordial labeling in the context of various graph operations is discussed by Vaidya and Barasara [7].

For any graph G, denote the number of vertices having label 1 as $v_f(1)$, the number of vertices having label 0 as $v_f(0)$, the number of edges having label 1 as $e_f(1)$ and the number of edges having label 0 as $e_f(0)$.

The product of two graphs is one of the important graph operations and mainly four different kinds of graph products are familiar. A detailed study on product graphs can be found in Hammack *et al.* [3].

The *cartesian product* $G \square H$ of two graphs $G = (V(G), E(G))$ and $E = (V(H), E(H))$ is a graph with the vertex set $V(G \square H) = V(G) \times V(H)$ specified by putting (u, v) adjacent to (u', v') if and only if (i) $u = u'$ and $vv' \in E(H)$, or (ii) $v = v'$ and $uu' \in E(G)$.

The *direct product (tensor product)* $G \times H$ of two graphs $G = (V(G), E(G))$ and $E = (V(H), E(H))$ is a graph with the vertex set $V(G \times H) = V(G) \times V(H)$ specified by putting (u, v) adjacent to (u', v') if and only if $uu' \in E(G)$ and $vv' \in E(H)$.

The *lexicographic product (graph composition)* $G[H]$ (also $G \circ H$) of two graphs $G = (V(G), E(G))$ and $E = (V(H), E(H))$ is a graph with the vertex set $V(G[H]) = V(G) \times V(H)$ specified by putting (u, v) adjacent to (u', v') if and only if (i) $uu' \in E(G)$, or (ii) $u = u'$ and $vv' \in E(H)$.

The graph product is called non-trivial, if both G and H have at least two vertices.

The present work is aimed to study edge product cordial labeling in the context of cartesian product, direct product and lexicographic product.

2 Main Results

Theorem 2.1. *The graph $P_m \Box P_n$ is not an edge product cordial graph.*

Proof. For the graph $P_m \Box P_n$, $|V(P_m \Box P_n)| = mn$ and $|E(P_m \Box P_n)| = 2mn - m - n$. We will consider following two cases.

Case 1: When n is even.

Subcase 1: When m is odd.

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to $\frac{2mn-m-n-1}{2}$ edges out of $2mn - m - n$ edges. The edges with label 0 will give rise at least $\frac{mn+m-1}{2}$ vertices with label 0 and at most $\frac{mn-m+1}{2}$ vertices with label 1 out of total mn vertices. Therefore $|v_f(0) - v_f(1)| = m - 1 > 1$. Consequently the graph is not edge product cordial.

Subcase 2: When m is even.

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to $\frac{2mn-m-n}{2}$ edges out of $2mn - m - n$ edges. The edges with label 0 will give rise at least $\frac{mn+m}{2}$ vertices with label 0 and at most $\frac{mn-m}{2}$ vertices with label 1 out of total mn vertices. Therefore $|v_f(0) - v_f(1)| = m > 1$. Consequently the graph is not edge product cordial.

Case 2: When n is odd.

Subcase 1: When $m \equiv 0 \pmod 4$

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to $\frac{2mn-m-n-1}{2}$ edges out of $2mn - m - n$ edges. The edges with label 0 will give rise at least $\frac{2mn+m}{4}$ vertices with label 0 and at most $\frac{2mn-m}{4}$ vertices with label 1 out of total mn vertices. Therefore $|v_f(0) - v_f(1)| = \frac{m}{2} > 1$. Consequently the graph is not edge product cordial.

Subcase 2: When $m \equiv 1 \pmod 4$

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to $\frac{2mn-m-n}{2}$ edges out of $2mn-m-n$ edges. The edges with label 0 will give rise at least $\frac{2mn+m+1}{4}$ vertices with label 0 and at most $\frac{2mn-m-1}{4}$ vertices with label 1 out of total mn vertices. Therefore $|v_f(0) - v_f(1)| = \frac{m+1}{2} > 1$. Consequently the graph is not edge product cordial.

Subcase 3: When $m \equiv 2 \pmod 4$

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to $\frac{2mn-m-n-1}{2}$ edges out of $2mn - m - n$ edges. The edges with label 0 will give rise at least $\frac{2mn+m+2}{4}$ vertices with label 0 and at most $\frac{2mn-m-2}{4}$ vertices with label 1 out of total mn vertices. Therefore $|v_f(0) - v_f(1)| = \frac{m+2}{2} > 1$. Consequently the graph is not edge product cordial.

Subcase 4: When $m \equiv 3 \pmod 4$

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to $\dfrac{2mn-m-n}{2}$ edges out of $2mn-m-n$ edges. The edges with label 0 will give rise at least $\dfrac{2mn+m+3}{4}$ vertices with label 0 and at most $\dfrac{2mn-m-3}{4}$ vertices with label 1 out of total mn vertices. Therefore $|v_f(0)-v_f(1)| = \dfrac{m+3}{2} > 1$. Consequently the graph is not edge product cordial.

Hence, $P_m \Box P_n$ is not an edge product cordial graph. □

Theorem 2.2. *The graph $C_m \Box C_n$ is not an edge product cordial graph.*

Proof. For the graph $C_m \Box C_n$, $|V(C_m \Box C_n)| = mn$ and $|E(P_m \Box P_n)| = 2mn$. Without loss of generality we assume that $m \leq n$. We will consider following two cases.

Case 1: When n is even.

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to mn edges out of $2mn$ edges. The edges with label 0 will give rise at least $\dfrac{mn}{2} + m$ vertices with label 0 and at most $\dfrac{mn}{2} - m$ vertices with label 1 out of total mn vertices. Therefore $|v_f(0) - v_f(1)| = 2m > 1$. Consequently the graph is not edge product cordial.

Case 2: When n is odd.

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to mn edges out of $2mn$ edges. The edges with label 0 will give rise at least $\dfrac{mn+m}{2}$ vertices with label 0 and at most $\dfrac{mn-m}{2}$ vertices with label 1 out of total mn vertices. Therefore $|v_f(0) - v_f(1)| = m > 1$. Consequently the graph is not edge product cordial.

Hence, $C_m \Box C_n$ is not an edge product cordial graph. □

Theorem 2.3. *The graph $P_m \times P_n$ is an edge product cordial graph.*

Proof. For the graph $P_m \times P_n$, $|V(P_m \times P_n)| = mn$ and $|E(P_m \times P_n)| = 2(m-1)(n-1)$. We will consider following two cases.

Case 1: When m or n is even.

The graph $P_m \times P_n$ is a disconnected graph with two components. Both the components are of the order $\dfrac{mn}{2}$ and the size $(m-1)(n-1)$. Assign label 1 to all the edges of one component and label 0 to remaining edges. As a result of this procedure we have the following:

$$e_f(1) = e_f(0) = (m-1)(n-1),$$
$$v_f(1) = v_f(0) = \frac{mn}{2}.$$

Case 2: When m and n are odd.

The graph $P_m \times P_n$ is a disconnected graph with two components. First component is of order $\left\lceil \dfrac{mn}{2} \right\rceil$ and second component is of the order $\left\lfloor \dfrac{mn}{2} \right\rfloor$ while the size of both the component is $(m-1)(n-1)$.

Assign label 1 to all the edges of component having order $\left\lceil \frac{mn}{2} \right\rceil$ and label 0 to remaining edges. As a result of this procedure we have the following:

$$e_f(1) = e_f(0) = (m-1)(n-1),$$
$$v_f(1) - 1 = v_f(0) = \frac{mn-1}{2}.$$

Thus in both the cases we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| = 0$.

Hence, $P_m \times P_n$ is an edge product cordial graph. □

Example 2.4. *The graph $P_4 \times P_7$ and its edge product cordial labeling is shown in figure 1.*

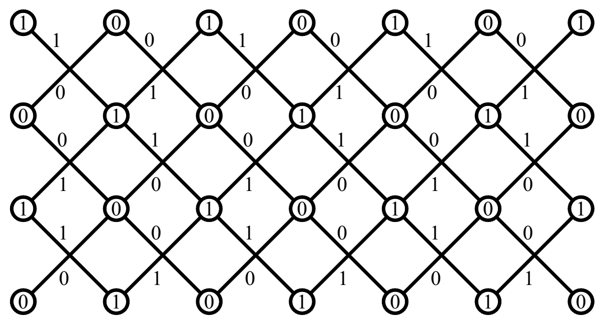

figure 1

Theorem 2.5. *The graph $C_m \times C_n$ is an edge product cordial graph for m and n even.*

Proof. For the graph $C_m \times C_n$, $|V(C_m \times C_n)| = mn$ and $|E(C_m \times C_n)| = 2mn$. For even m and n, the graph $C_m \times C_n$ is a disconnected graph with two components. Both the components are of the order $\frac{mn}{2}$ and the size mn. Assign label 1 to all the edges of one component and label 0 to remaining edges. As a result of this procedure we have the following:

$$e_f(1) = e_f(0) = mn,$$
$$v_f(1) = v_f(0) = \frac{mn}{2}.$$

Thus we have $|v_f(0) - v_f(1)| = 0$ and $|e_f(0) - e_f(1)| = 0$. Hence, $C_m \times C_n$ is an edge product cordial graph for m and n even. □

Example 2.6. *The graph $C_4 \times C_6$ and its edge product cordial labeling is shown in figure 2. Here grey edges are labeled with 1 and black edges are labeled with 0.*

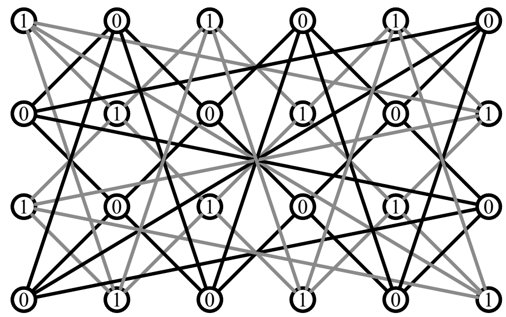

figure 2

Theorem 2.7. *The graph $C_m \times C_n$ is not an edge product cordial graph for m or n odd.*

Proof. For the graph $C_m \times C_n$, $|V(C_m \times C_n)| = mn$ and $|E(C_m \times C_n)| = 2mn$. Without loss of generality we assume that $m \leq n$. We will consider following three cases.

Case 1: When $m = 3$ and $n = 3$.

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to 9 edges out of 18 edges. The edges with label 0 will give rise at least 6 vertices with label 0 and at most 3 vertices with label 1 out of total 9 vertices. Therefore $|v_f(0) - v_f(1)| = 3$. Consequently the graph is not edge product cordial.

Case 2: When m is odd or n is odd but not both.

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to mn edges out of $2mn$ edges. The edges with label 0 will give rise at least $\frac{mn+4}{2}$ vertices with label 0 and at most $\frac{mn-4}{2}$ vertices with label 1 out of total mn vertices. Therefore $|v_f(0) - v_f(1)| = 4$. Consequently the graph is not edge product cordial.

Case 3: When both m and n are odd.

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to mn edges out of $2mn$ edges. The edges with label 0 will give rise at least $\frac{mn+5}{2}$ vertices with label 0 and at most $\frac{mn-5}{2}$ vertices with label 1 out of total mn vertices. Therefore $|v_f(0) - v_f(1)| = 5$. Consequently the graph is not edge product cordial.

Hence, $C_m \times C_n$ is not an edge product cordial graph for m or n odd. □

Theorem 2.8. *The graph $P_n[P_2]$ is not an edge product cordial graph.*

Proof. For the graph $P_n[P_2]$, $|V(P_n[P_2])| = 2n$ and $|E(P_n[P_2])| = 5n-4$. In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to $\left\lfloor \frac{5n-4}{2} \right\rfloor$ edges out of $5n-4$ edges.

The edges with label 0 will give rise at least $n+1$ vertices with label 0 and at most $n-1$ vertices with label 1 out of total $2n$ vertices. Therefore $|v_f(0) - v_f(1)| = 2$. Consequently the graph is not edge product cordial. Hence, $P_m[P_2]$ is not an edge product cordial graph. □

Theorem 2.9. *The graph $C_n[P_2]$ is not an edge product cordial graph.*

Proof. For the graph $C_n[P_2]$, $|V(C_n[P_2])| = 2n$ and $|E(C_n[P_2])| = 5n$. In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to $\left\lfloor \frac{5n}{2} \right\rfloor$ edges out of $5n$ edges. The edges with label 0 will give rise at least $n+2$ vertices with label 0 and at most $n-2$ vertices with label 1 out of total $2n$ vertices.

Therefore $|v_f(0) - v_f(1)| = 4$. Consequently the graph is not edge product cordial. Hence, $C_m[P_2]$ is not an edge product cordial graph. □

3 Concluding Remarks

We have investigated edge product cordial labeling for the larger graph obtained by means of three graph products, namely cartesian product, direct product and lexicographic product.

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Simulation of objective function for training of new hidden units in constructive Neural Networks

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Abstract : The present research article represent the mathematical analysis of objective function for training new hidden units in constructive algorithms for multi-layer feed-forward networks. Neural research, now days, is highly attractive wing under research community which may lead the development of some hidden prospects by using mathematical modeling which involve the design of neurons network. The network size is highly important for neural network. Small network as well as large network size cannot be learned very well, but there is an optimum size for which the neural network can be involved for good results. Constructive algorithms started with a small network size and then grow additional hidden units until a satisfactory solution is found. A network, having n-1 hidden units, is directly connected to the output unit, which is modeled as $f_{n-1} = \sum_{j=1}^{n-1} \beta_j g_j$ and $e_{n-1} = f - f_{n-1}$ is the residual error function for current network with $n-1$ hidden units. A new hidden unit is added under a process in input as a linear combination of g_n with the current network $f_{n-1} + \beta_n g_n$ which governed the minimum residual error $\|e_n\|$ in the output process by keeping g_n fixed and adjusted value of β_n so as to minimize residual error. The function to be optimized during input training is $\frac{\langle e_{n-1}, g_n\rangle^2}{\|g_n\|^2}$ and corresponding objective function is

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$s_1 = \frac{(\Sigma_p E_p H_p)^2}{\|g_n\|^2}$, where H_p is the activation function of the new hidden unit and E_p is the corresponding residual error before this new hidden unit is added.

Keywords : constructive algorithms, hidden units, feed forward networks.

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1 Introduction

Many neural network models have been proposed for pattern classification, function approximation and regression problems. Optimization of neural network topology has been one of the most important problems since neural network came in front as a method for prediction and association. Number of heuristic formula for determining the numbers of hidden units were developed and some algorithms for structure optimization were suggested, such as cascading, pruning and others [1].

The network size is very important for neural network, small network as well as large network size can not learn very well but there is an optimum size for which the neural network can be involved the good result. Constructive algorithm finds an appropriate network size automatically for a given application and optimizes the set of network weight. Constructive algorithms start with a small network size and then grow additional hidden units and weights until a satisfactory solution is found.Constructive algorithm is straight forward to specify initial network. Initial network means the smallest possible network to start with has no hidden units.

Constructive algorithms always search for small network solution first so constructive algorithm is more computationally economical because smaller network are more efficient in forward computation. The neural network with fixed architecture is only trained once, but in constructive methods, every time the architecture is changed and the training must be repeated. The computational cost of repeated training is high, so some techniques as weight freezing, re-training the whole network continuing from previous weights and re-training the whole network from scratch are applied to reduce both time and space complexity. To improve computational complexity, in constructive algorithm, it is assumed that the hidden units already exiting in the network and this useful in modeling part of the target function.

Hence the weights feeding into these hidden units fixed (input weight freezing) and allow only the weights connected to the new hidden unit and output hidden units to vary. The number of weights to be optimized, and the time and space requirements for each iteration can thus be greatly reduce. This reduction is especially significant when there are already many hidden units installed in the network. So neural networks are now days, being used to solve a whole range of problems, most common case studies include modeling hitherto intractable processes, designing complex feed-back control signal, building meta-models for posterior optimization or detecting device faults [2]. Cascade correlation learning algorithm starts with a small network. The key idea of for cascade-correlation type constructive neural network algorithms is to split the pool of candidate hidden units into two groups. The candidate, which is installed in the active network, either becomes part of the current deepest hidden layer or adds a new hidden layer to the network [7]. Adding and training new hidden units as and when required to form a multilayer network. It uses the Quickprop algorithm to manage the step-size problem that occurs in back propagation algorithm. Second order method used in Quickprop algorithm to updates the weights. The algorithm initially starts with some input and output units connected by an adjustable weight. There are no hidden units yet.

We now freeze the weights of the hidden units and trained the connections at the output. The output weights can be trained by Quickprop algorithm as it converges faster with no hidden units. We stop the training cycle when significant error reduction has been achieved. In case of further reducing the error we may add hidden units and repeat these steps to achieve a small error. Now we adjust the input weight of the candidate unit. So that we can minimize the sum over all the output units. A collection of candidate units having different initial weights being trained simultaneously can also be used. The unit with the best correlation can be added to the network [5].

The puzzle of the human brain is as old as human history. By day-today experience we accumulate and associate all advances in human knowledge. The design of network based on intelligent is generally gives the network learning and the tool which is developed for learning is called neural network [3]. Artificial neural network has a lot of important capabilities such as learning from data, generalization, working with unlimited number of variables. Neural networks may be used as a direct substitute for autocorrelation, multivariable regression, linear regression, trigonometric and other statistical analytic techniques [4]. Successful application examples show that human diagnostic capabilities are significantly worse than the neural diagnostic system. A constructive neural network algorithm with backpropagation offers an approach for incremental construction of near- minimal neural network architectures for pattern classification. The algorithm starts with minimal number of hidden units in a single hidden layer, additional units are added to the hidden layer one at a time to improve the accuracy of the network and to get an optimal size of a neural network.

For building and training single hidden layer neural network a simultaneous perturbation training algorithm is used, in this method all hidden neurons are used to model the relationships between input data and modal residuals. A sigmoid hidden neuron is added to the single layer of hidden neurons after a period of training when the error has stopped decreasing below a threshold value. After the addition of the new hidden neuron, the whole network is again trained with simultaneous perturbation. In training, perturbed value to the connection weights are changed to detrap the local minima [10].The role of adaptive sigmoidal activation function has been verified in constructive neural networks for better generalization performance and lesser training time [6]. Constructive neural networks are a collection of a group of methods, which enable the network architecture to be constructed along with the training process. Constructive algorithms that construct feed-forward architecture for regression problems.

A general constructive approach for training neural networks in classification problem is used to construct a particular connectionist model, named switching neural network (SNN), based on the conversion of the original problem in a Boolean lattice domain [8]. In general a constructive algorithm has two integral components pre-specified network growing strategy and local optimization technique for updating weights during learning. In a constructive adaptive neural network, the error signals during the learning process improve the input-side training effectiveness and efficiency, and obtain better generalization performance capabilities [11].

2 Design of the Objective Function

Constructive algorithms are supervised learning algorithms for neural network. They start individual neural network with a small architecture, small number of hidden layers, nodes and connection, and then add hidden nodes and weights incrementally until a satisfactory solution is found. Constructive algorithms have been applied to benchmark problems, which are popular both in case of machine learning and neural network community [9].

A network, having n-1 hidden units directly connected to the output unit, implements the function

$$f_{n-1} = \sum_{j=1}^{n-1} \beta_j g_j$$

Where g_j represent the function implemented by the j_{th} hidden unit. These g_j may only be indirectly connected to the input units through intermediate hidden units, and the network is not restricted to having only one single hidden layer. Moreover, $e_{n-1} \equiv f - f_{n-1}$ is the residual error function for the current network with $n-1$ hidden units.

Addition of a new hidden unit proceeds in two steps:

Input training: find β_n and g_n such that the resultant linear combination of g_n with the current network, i.e. $f_{n-1} + \beta_n g_n$, gives minimum residual error $\|e_n\|$.

Output training: keeping $g_1, g_2, ...g_n$ fixed, adjust the value of $\beta_1, \beta_2, ...\beta_n$ so as to minimize the residual error.

Derivation for objective function

For a fixed $g(\|g\| \neq 0)$, the expression $\|f - (f_{n-1} + \beta g)\|$ achieves its minimum iff $\beta = \beta^* = \frac{\langle e_{n-1}, g\rangle}{\|g^2\|}$. Moreover, with β_n^* and β^* as define above, $\|f - (f_{n-1} + \beta_n^* g_n)\| \leq \|f - (f_{n-1} + \beta^* g)\|$ $\forall g$, iff $\frac{\langle e_{n-1}, g_n\rangle^2}{\|g_n\|^2} \geq \frac{\langle e_{n-1}, g\rangle^2}{\|g\|^2}$ $\forall g$.

Case:1 When all elements are of unit norm

$$\begin{aligned}\Delta g &\equiv \|f - f_{n-1}\|^2 - \|f - (f_{n-1} + \beta g)\|^2 \\ &= 2\langle f - f_{n-1}, \beta g\rangle - \beta^2\langle g, g\rangle\end{aligned}$$

Put $f - f_{n-1} = e_{n-1}$ then we find $\Delta g = 2\beta\langle e_{n-1}, g\rangle - \beta^2\|g\|^2$

Taking $\|g\| = 1$ the expression will be $\Delta g = 2\langle e_{n-1}, g\rangle - \beta^2$. Adding and subtracting $\langle e_{n-1}, g\rangle^2$ then we find the expression $\Delta g = \langle e_{n-1}, g\rangle^2 - (\langle e_{n-1}, g\rangle - \beta)^2$. So for a fixed g the stated expression is minimized iff $\beta = \beta^* = \langle e_{n-1}, g\rangle$ with $\Delta_{max}(g) = \langle e_{n-1}, g\rangle^2$. This shows that the stated expression is minimized when $\Delta_{max}(g)$ is maximized over all g.

Case:2 When the assumption of unit norm is dropped, then the stated expression is minimized iff $\beta = \beta^* = \frac{\langle e_{n-1}, g\rangle}{\|g\|^2}$ with $\Delta_{max}(g) = \frac{\langle e_{n-1}, g\rangle}{\|g\|^2}$. This result suggests that the objective function to be optimized during input training is $\frac{\langle e_{n-1}, g\rangle^2}{\|g\|^2}$. The above function can only be calculated when the exact functional form of e_{n-1} is available, which is obviously impossible as the true f is unknown. A consistent estimate of above function using information from the training set is $\frac{(\frac{1}{N}\Sigma_p E_p H_p)^2}{\frac{1}{N}\Sigma_p H_p)^2}$, where H_p is the activation of the new hidden unit for the pattern p and E_p is the corresponding residual error before this new hidden unit is added. Dropping the factor $\frac{1}{N}$ which is common for all candidate hidden unit function, we obtain the objective function- $S = \frac{(\Sigma_p E_p H_p)^2}{\Sigma_p H_p)^2}$.

3 Conclusions

We have studied the mathematical analysis of the objective function for training new hidden units in constructive algorithm. The objective function to be optimized during input training is $\frac{\langle e_{n-1}, g_n\rangle^2}{\|g_n\|^2}$ which can be used for training of new hidden units.

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Graceful Labelling of Supersubdivision of Ladder

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Abstract : In this paper we prove that supersubdivisions of ladders are graceful.

Keywords : graceful labelling, subdivision of graphs, supersubdivision of graphs.

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1 Introduction

Let G be a graph with q edges. A graceful labelling of G is an injection from the set of its vertices to the set $\{0, 1, 2, \ldots, q\}$ such that the values of the edges are all integers from 1 to q, the value of an edge being the absolute value of the difference between the integers attributed to its end vertices.

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Recently G. Sethuraman and P. Selvaraju [5] have introduced a new method of construction called supersubdivision of a graph and showed that arbitrary supersubdivisions of paths are graceful. They have posed two open problems:

Problem 1.1. Is there any graph different from paths whose arbitrary supersubdivisions are graceful?

Problem 1.2. Is it true that every connected graph has at least one supersubdivision which is graceful?

We work on these problems and find that supersubdivisions of ladders are graceful. The ladder graph L_n is defined by $L_n = P_n \times K_2$ where P_n is a path with $\times$ denotes the cartesian product. L_n has $2n$ vertices and $3n-2$ edges. In the complete bipartite graph $K_{2,m}$ we call the part consisting of two vertices, the 2-vertices part of $K_{2,m}$ and the part consisting of m vertices the m-vertices part of $K_{2,m}$.

Let G be a graph with n vertices and t edges. A graph H is said to be a subdivision of G if H is obtained by subdividing every edge of G exactly once. H is denoted by $S(G)$. A graph H is said to be a supersubdivision of G if H is obtained by replacing every edge e_i of G by the complete bipartite graph $K_{2,m}$ for some positive integer m in such a way that the ends of e_i are merged with the two vertices part of $K_{2,m}$ after removing the edge e_i from G.

A supersubdivision H of a graph G is said to be an arbitrary supersubdivision of the graph G if every edge of G is replaced by an arbitrary $K_{2,m}$ (m may vary for each edge arbitrarily). In this paper we prove that supersubdivisions of ladders are graceful.

2 Main Results

Let L_n be a ladder. A supersubdivision of L_n is denoted by $SS(L_n)$.

Theorem 2.1. *$SS(L_n)$ with each edge replaced by $K_{2,m}$ is graceful.*

Proof. Let $G = SS(L_n)$ where every edge of L_n is replaced by $K_{2,m}$. G has $2n + m(3n-2)$ vertices and $2m(3n-2)$ edges. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices of L_n. Let $u_iu_{i+1}, i = 1, 2, \ldots, n-1$. $v_iv_{i+1}, i = 1, 2, \ldots, n-1$ and $u_iv_i, i = 1, 2, \ldots, n$ be the edges of L_n.

Let $u_{i,i+1}^{(k)}, k = 1, 2, \ldots, m$ be the vertices of the m vertices part of the bipartite graph $K_{2,m}$ merged with the edge $u_iu_{i+1}, i = 1, 2, \ldots, n-1$. Let $v_{i,i+1}^{(k)}, k = 1, 2, \ldots, m$ be the vertices of the m vertices part of the bipartite graph $K_{2,m}$ merged with the edge $v_iv_{i+1}, i = 1, 2, \ldots, n-1$.

Let $w_i^{(k)}, k = 1, 2, \ldots, m$ be the vertices of the m vertices part of $K_{2,m}$ merged with the edge $u_iv_i, i = 1, 2, \ldots, n$. Naming of the vertices is as shown in Figure 1.

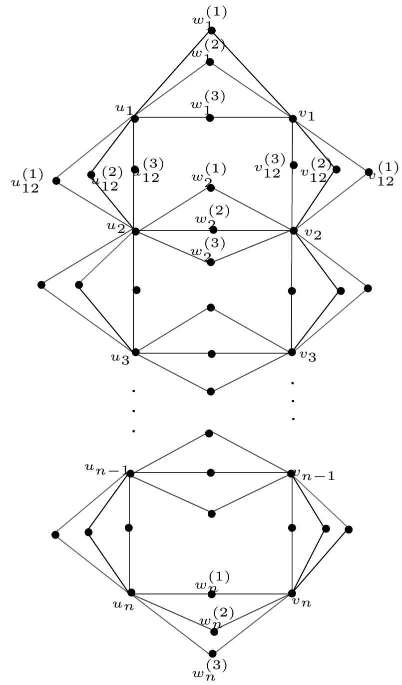

Figure 1: $SS(L_n)$

Define the following functions $\eta : N \to N$ by

$$\eta(i) = \begin{cases} 2i-1 & \text{if } i \text{ is even} \\ 2(i-1) & \text{if } i \text{ is odd} \end{cases}$$

$\gamma : N \to N$ by

$$\gamma(i) = \begin{cases} 2(i-1) & \text{if } i \text{ is even} \\ 2i-1 & \text{if } i \text{ is odd} \end{cases}$$

$\alpha : N \to N$ by

$$\alpha(k) = \begin{cases} 6r & \text{if } k = 3r+1 \\ 6r+1 & \text{if } k = 3r+2 \\ 6r+2 & \text{if } k = 3r+3 \end{cases}$$

Case (i) $m \equiv 0(mod\ 3)$.

Let $m = 3p$, where p is a positive integer. Define $f : V \to \{0, 1, 2, \ldots, q\}$ where $q = 2m(3n-2)$ as follows.

$$f(u_i) = \eta(i), i = 1, 2, \ldots, n$$

$$f(v_i) = \gamma(i), i = 1, 2, \ldots, n.$$

For $k = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n-1$

$$\text{define } f(u^{(k)}_{i,i+1}) = \begin{cases} 2m(3n-2) - (i-1)(18p-2) & \\ +\alpha(k) - (12p-4) & \text{if } i \text{ is odd} \\ 2m(3n-2) - (i-2)(18p-2) & \\ -2(m-k) - (30p-3) & \text{if } i \text{ is even.} \end{cases}$$

For $k = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n-2$ define $f(v^{(k)}_{i,i+1}) = f(u^{(k)}_{i+1,i+2}) + (18p-2)$.

For $k = 1, 2, \ldots, m$ define $f(v^{(k)}_{n-1,n}) = f(u^{(k)}_{n-2,n-1}) - (18p-2)$.

For $k = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n-1$ define $f(w^{(k)}_i) = 2m(3n-2) - (i-1)(18p-2) - 2(m-k)$.

For $k = 1, 2, \ldots, m$ define $f(w^{(k)}_n) = 2(n+k-1)$.

Example 2.1. Graceful labelling of $SS(L_4)$ where each edge of L_4 is replaced by $K_{2,6}$.

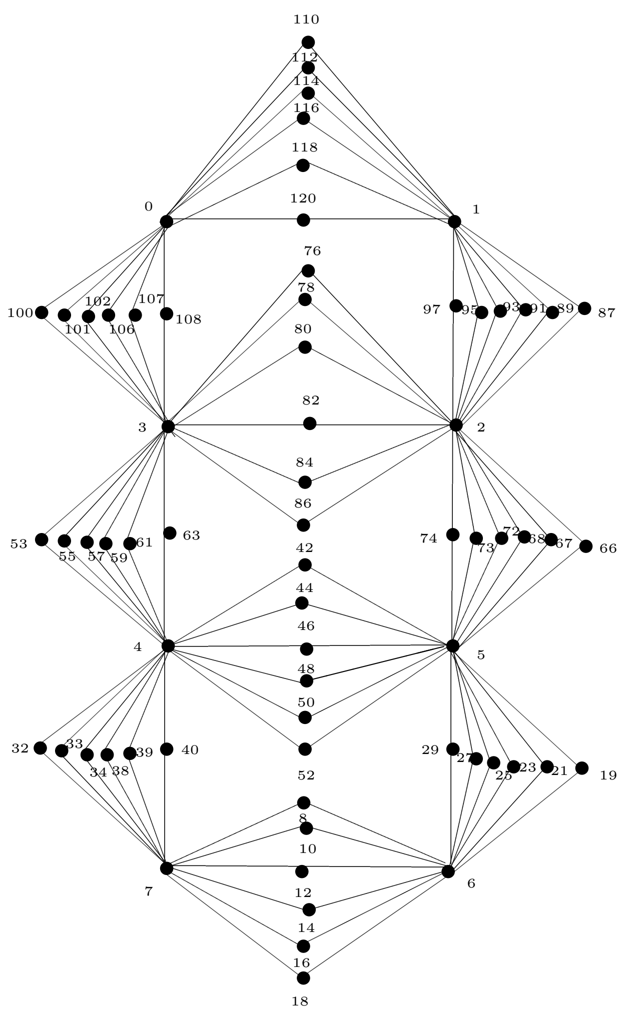

Figure 2

Case ii $m \equiv 1(mod\ 3)$.

Let $m = 3p+1$ where p is a nonnegative integer. Define $f : V \to \{0, 1, 2, \ldots, q\}$ where $q = 2m(3n-2)$ as follows.

$$f(u_i) = \eta(i), i = 1, 2, \ldots, n$$
$$f(v_i) = \gamma(i), i = 1, 2, \ldots, n.$$

When i is even, define $f(u^{(k)}_{i,i+1}) = 2m(3n - 2) - (i - 2)(18p + 4) - 2(m - k) - (30p + 7)$ for $k = 1, 2, 3, \ldots, m$. When i is odd, define

$$f(u^{(k)}_{i,i+1}) = \begin{cases} 2m(3n-2) - (i-1)(18p+4) & \\ +\alpha(k) - 12p & \text{if } k = 1, 2, \ldots, m-1 \\ 2m(3n-2) - (i-1)(18p+4) & \\ -6p & \text{if } k = m \end{cases}$$

For $i = 1, 2, \ldots, n - 2$, define $f(v^{(k)}_{i,i+1}) = f(u^{(k)}_{i+1,i+2}) + (18p + 4)$ for $k = 1, 2, \ldots, m$.

Define $f(v^{(k)}_{n-1,n}) = f(u^{(k)}_{n-2,n-1}) - (18p + 4)$ for $k = 1, 2, \ldots, m$.

For $i = 1, 2, 3, \ldots, n - 1$, define

$$f(w^{(k)}_i) = \begin{cases} 2m(3n-2) - (i-1)(18p+4) & \\ -(6p+1) & \text{if } k = 1 \\ 2m(3n-2) - (i-1)(18p+4) & \\ -2(m-k) & \text{if } k = 2, 3, \ldots, m \end{cases}$$

For $k = 1, 2, \ldots, m$, define $f(w^{(k)}_n) = 2(n + k - 1)$.

Example 2.2. Graceful labelling of $SS(L_5)$ where each edge of L_5 is replaced by $K_{2,4}$.

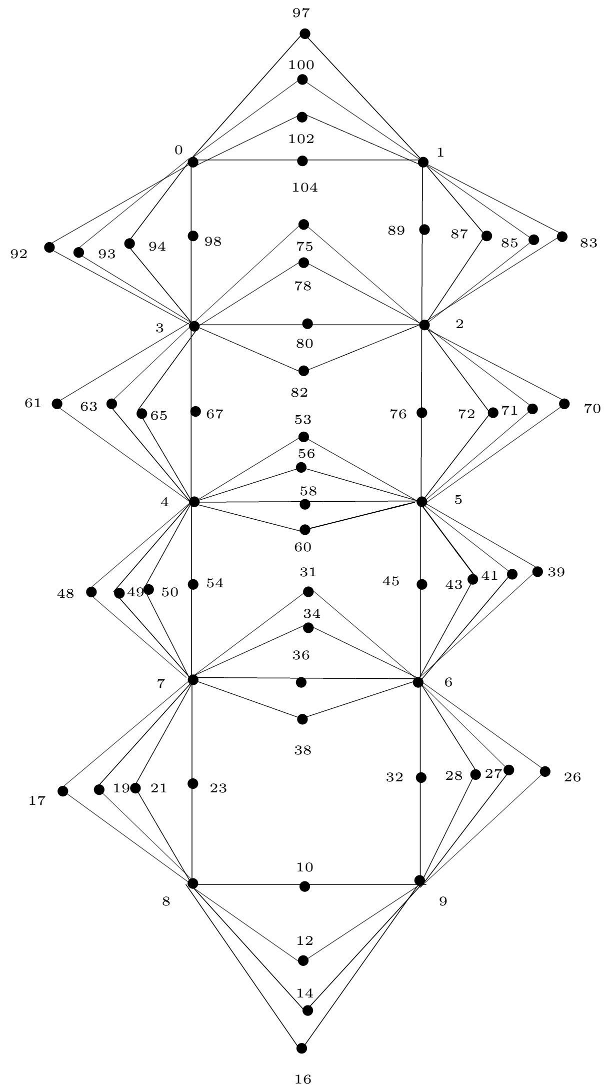

Figure 3

Note. In the above, taking $p = 0$ we obtain subdivision of a ladder and we have a graceful labelling of the subdivision of ladders, as a deduction from our labelling of vertices. This gives another graceful labelling for subdivision of ladders established by KM. Kathiresan [3].

Example 2.3. Graceful labelling of $S(L_6)$.

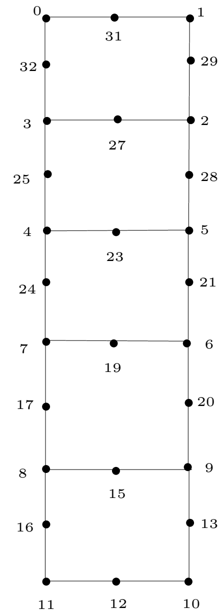

Figure 4

Case (iii) $m \equiv 2(mod\ 3)$.

Let $m = 3p+2$ where p is a nonnegative integer. Define $f : V \to \{0, 1, 2, \ldots, q\}$ where $q = 2m(3n-2)$ as follows.

$$f(u_i) = \eta(i), i = 1, 2, 3, \ldots, n$$

$$f(v_i) = \gamma(i), i = 1, 2, 3, \ldots, n.$$

When i is even, define $f(u_{i,i+1}^{(k)}) = 2m(3n-2) - (i-2)(18p+10) - 2(m-k) - (30p+17)$ for $k = 1, 2, \ldots, m$.

When i is odd, define $$f(u_{i,i+1}^{(k)}) = \begin{cases} 2m(3n-2) - (i-1)(18p+10) & \\ +\alpha(k) - 4(3p+1) & \text{if } k = 1, 2, \ldots, m-2 \\ 2m(3n-2) - (i-1)(18p+10) & \\ -2(3p+2) & \text{if } k = m-1 \\ 2m(3n-2) - (i-1)(18p+10) & \\ -6p & \text{if } k = m \end{cases}$$

For $i = 1, 2, \ldots, n-2$, define $f(v_{i,i+1}^{(k)}) = f(u_{i+1,i+2}^{(k)}) + (18p+10)$.

For $k = 1, 2, \ldots, m$, define $f(v_{n-1,n}^{(k)}) = f(u_{n-2,n-1}^{(k)}) - (18p+10)$.

For $k = 1, 2, \ldots, m$, define $f(w_n^{(k)}) = 2(n+k-1)$.

For $k = 1, i = 1, 2, \ldots, n-1$, define $f(w_i^{(k)}) = 2m(3n-2) - (i-1)(18p+10) - (6p+5)$.

For $k = 2, i = 1, 2, \ldots, n-1$, define $f(w_i^{(k)}) = 2m(3n-2) - (i-1)(18p+10) - (6p+1)$.

For $k = 3, 4, \ldots, m$ and $i = 1, 2, \ldots, n-1$, define $f(w_i^{(k)}) = 2m(3n-2) - (i-1)(18p+10) - 2(m-k)$.

Example 2.4. Graceful labelling of $SS(L_5)$ where each edge of L_5 is replaced by $K_{2,5}$.

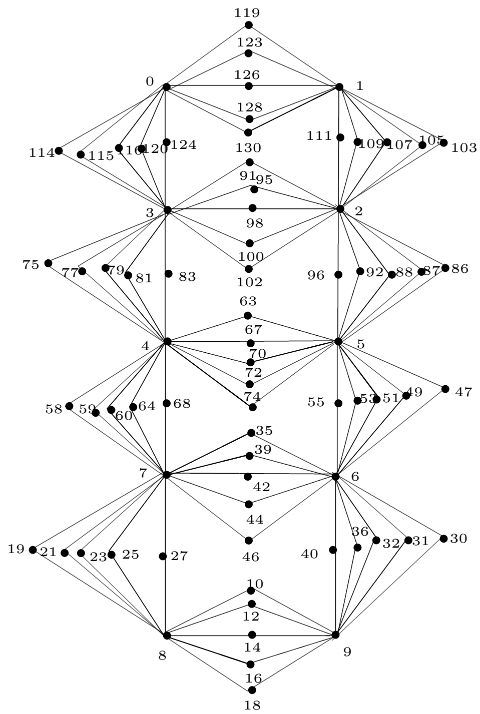

Figure 5

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Minimum 2-Edge Connected Spanning Subgraph of Certain Interconnection Networks

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Abstract : Given an undirected graph, finding a minimum 2-edge connected spanning subgraph is NP-hard. We solve the problem for silicate network, brother cell and sierpiński gasket rhombus.

Keywords : silicate network; brother cell; sierpiński gasket rhombus.

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1 Introduction

The study of connectivity in graph theory has important applications in the areas of network reliability and network design. In fact, with the introduction of fiber optic technology

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AMS Subject Classification: 05C40

in telecommunication, designing a minimum cost survivable network has become a major objective in telecommunication industry. Survivable networks have to satisfy some connectivity requirements, this means that they are still functional after the failure of certain links [5]. As pointed out in [5, 9], the topology that seems to be very efficient is the network that survives after the loss of $k-1$ or less edges, for some $k \geq 2$, where k depends on the level of reliability required in the network [9]. In this paper, we concentrate on the minimum 2-edge connected spanning subgraph. A connected graph $G = (V, E)$ is said to be 2-edge connected if $|V| \geq 2$ and the deletion of any set of < 2 edges leaves a connected graph. The minimum 2-edge connected spanning subgraph (2-ECSS) problem is defined as follows: Given a 2-edge connected graph G, find efficiently a spanning subgraph $S(G)$ which is also 2-edge connected and has a minimum number of edges. We denote the number of edges in a graph G by $\varepsilon(G)$ and the edges of minimum 2-edge connected spanning subgraph of G by $\varepsilon(S(G))$.

Kuller and Raghavachari [12] presented the first algorithm which, for all k, achieves a performance ratio smaller than a constant which is less than two. They proved an upper bound of 1.85 for the performance ratio of their algorithm. Cristina G. Fernandes [7] improved their analysis, proving that the performance ratio of algorithm [13] is smaller than 1.7 for large enough k, and that it is at most 1.75 for all k. Cherian et.al [6] gave an approximation algorithm for minimum size 2-ECSS problem where an ear decomposition is used to construct a feasible 2-ECSS. The depth-first search algorithm was used to present a 3/2 approximation algorithm for the minimum size 2-ECSS problem in which a notion called tree carving is used [13]. An approximation for finding a smallest 2-edge connected subgraph containing a specified spanning tree was studied by Hiroshi Nagamochi [8]. The sufficient conditions for a graph to be perfectly 2-edge connected was given by Ali Ridha Mahjoub [2]. Woonghee [15] devised an algorithm for r-regular, r-edge connected graphs. For cubic graphs, results of [11] imply a new upper bound on the integrality gap of the linear programming formulation for the 2-edge connectivity problem. Even though there are numerous results and discussions on minimum 2-edge connected spanning subgraph problem, most of them deal only with approximation results. According to the literature survey, the minimum 2-edge connected spanning subgraph problem is not solved for an interconnection network. In this paper we derive an exact number of edges of minimum 2-edge connected spanning subgraph of silicate network, brother cell and sierpiński gasket rhombus.

2 Silicate Network

Lemma 2.1. *[1] If one end of every edge of a graph G is of degree 2 then no proper spanning subgraph of G is 2-edge connected.*

Consider a honeycomb network $HC(r)$ of dimension r. Place silicon ions on all the vertices of $HC(r)$. Subdivide each edge of $HC(r)$ once. Place oxygen ions on the new vertices. Introduce $6r$ new pendant edges one each at the 2-degree silicon ions of $HC(r)$ and place oxygen ions at the pendent vertices. See Figure 1(a). With every silicon ion associate the three adjacent oxygen ions and form a tetrahedron as in Figure 1(b). The resulting network is a silicate network of dimension r, denoted $SL(r)$. The diameter of $SL(r)$ is $4r$. The graph in Figure 1(b) is a silicate network of dimension two. The 3-degree oxygen nodes of silicates are called boundary nodes. In Figure 1(b), c_1, c_2, $\cdots$, c_{12} are boundary nodes SL_2.

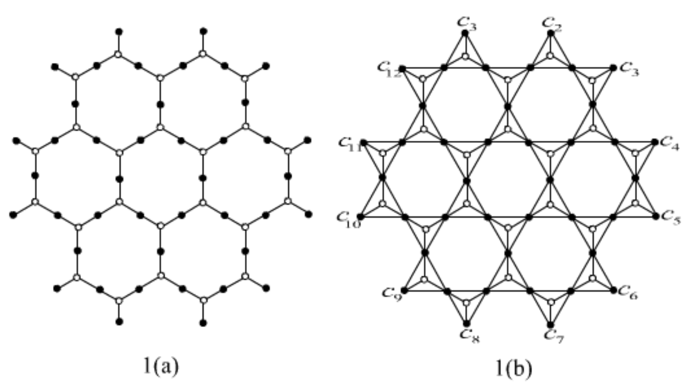

Figure 1: Silicate Network $SL(2)$

When we delete all the silicon nodes from a silicate network we obtain a new network which we shall call as an Oxide Network [14]. See Figure 2(a). An r-dimensional oxide network is denoted by $OX(r)$. By [14], there are r edge disjoint symmetric cycles in $OX(r)$ which are also vertex disjoint cycles. Let them be x_1, x_2, ..., x_r. See Figure 2(b). The number of edges in $x_i, 1 \leq i \leq r$ is $18i - 6$.

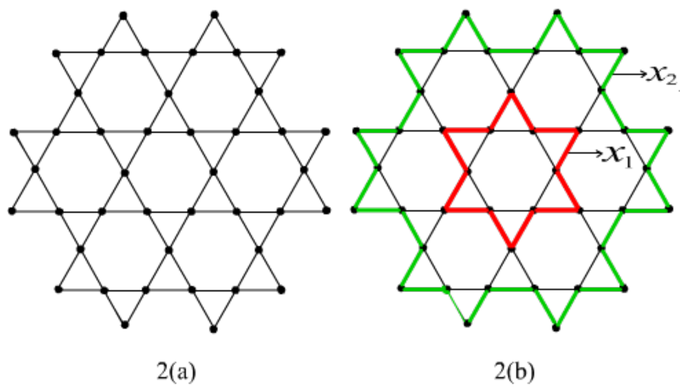

Figure 2: Oxide Network $OX(2)$

Theorem 2.2. *Let $OX(r), r \geq 2$ be an r-dimensional oxide network. Then* $\varepsilon(S(OX(r))) = \varepsilon(x_1) + \varepsilon(x_2) - 1 + \cdots + \varepsilon(x_r) - 1 + 2(r-1)$.

Proof. Let us prove the theorem by induction on r. When $r = 2$, there are r=2 edge disjoint cycles x_1 and x_2 in $OX(2)$. keeping x_1 and x_2, removing all the edges, we get a disconnected oxide network with 2-edge disjoint cycles x_1 and x_2 in $OX(2)$. See Figure 3(a).

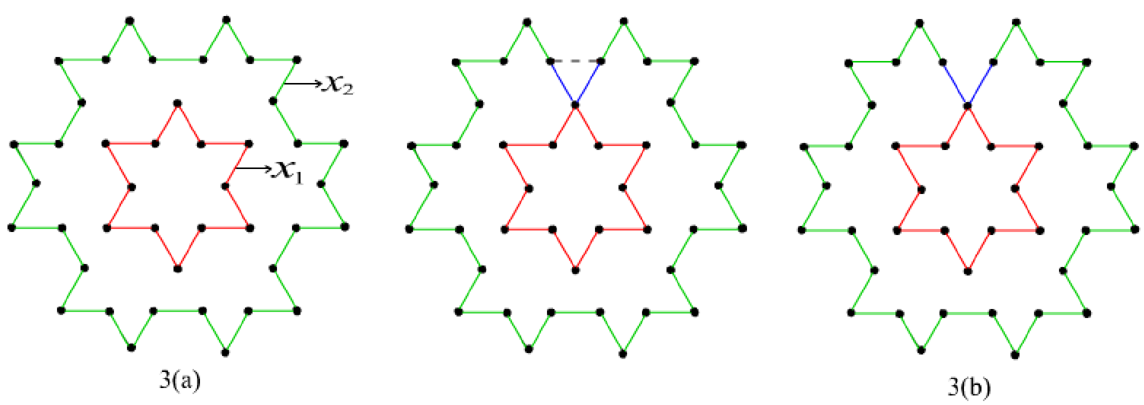

Figure 3: $\varepsilon(S(OX(r=2))) = 12 + 29 + 2 = 43$.

Adding 2 edges from a boundary vertex of x_1 to two non boundary adjacent vertices of x_2 and deleting the edge between those non boundary vertices of x_2 [edge to be removed is shown in dashed line], we get a minimum 2-edge connected spanning subgraph. See Figure 3(b). This is minimum because by Lemma 2.1, deleting any single edge gives no 2-edge connected spanning subgraph. The number of edges in x_1 and x_2 are $18(1) - 6$ and $18(2) - 6$. Hence $\varepsilon(S(OX(r=2))) = 12 + 29 + 2 = \varepsilon(x_1) + \varepsilon(x_2) - 1 + 2(r-1)$. Thus the result is true for $r = 2$.

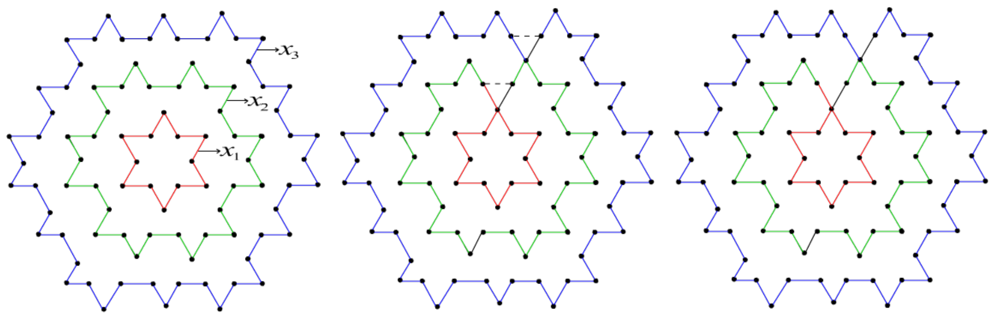

Figure 4: $\varepsilon(S(OX(r = 3))) = 12 + 29 + 47 + 4 = 92$.

We assume that the result is true for $r = k$. When $r = k + 1$, there are $r = k + 1$ edge disjoint cycles x_1, x_2, $\cdots$, x_{k+1}. Adding 2 edges from a boundary vertex of x_i, $1 \leq i \leq k$ to two non boundary adjacent vertices of x_{i+1}, $1 \leq i \leq k$ and deleting the edges between those non boundary vertices of x_2, x_3, $\cdots$, x_{k+1}, we get a minimum 2-edge connected spanning subgraph. Hence $\varepsilon(S(OX(r = k + 1))) = 18(1){-}6 - 1 + 18(2){-}6 - 1 + \cdots + 18((k{+}1)) - 6 - 1 + 2k$ $= \varepsilon(x_1) + \varepsilon(x_2) - 1 + \cdots + \varepsilon(x_{k+1}) - 1 + 2((k + 1) - 1)$. □

3 Sierpinski Gasket Rhombus

Definition 3.1. *[4] A sierpiński Gasket Rhombus of level r [denoted by SR_r] is obtained by identifying the edges in two Sierpinski Gasket S_r along one of their side. For the definition of sierpiński Gasket, refer[10].*

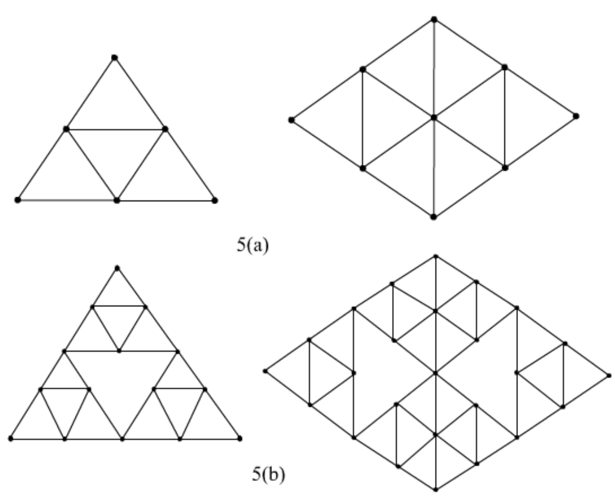

Figure 5:(a) S_2 and SR_2 and (b) S_3 and SR_3

The sierpiński Gasket graphs S_r has 3^r edges [14]. From the Definition 3.1, sierpiński Gasket

Rhombus SR_r consists two copies of sierpiński Gasket graph S_r and identifying the edges of two sierpiński Gasket graphs S_r along one of their side, 2^{r-1} edges are shared by both S_r. Therefore the number of edges in SR_r is $2{\times}3^r - 2^{r-1}$.

Theorem 3.2. *[1] Let $S_r, r \geq 3$ be the r dimensional Sierpiński gasket graph. Then $\varepsilon(S(S_r)) = 2 \times 3^{r-1}$.*

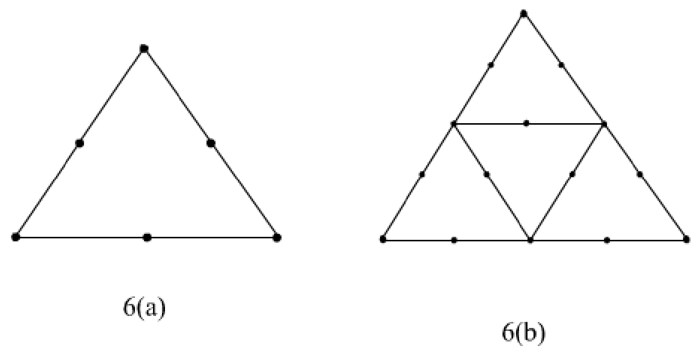

Figure 6: (a) $\varepsilon(S(S_2)) = 6$ and (b) $\varepsilon(S(S_3)) = 18$.

Theorem 3.3. *Let $SR_r, r \geq 2$ be the r dimensional sierpiński Gasket Rhombus. Then $\varepsilon(S(SR_r)) = 2(2 \times 3^{r-1}) - 2^{r-1}$.*

Proof. We prove this theorem by induction on r. When $r = 2$, SR_2 contains 2 copies of S_2 and has $2 \times 3^2 - 2^{2-1}$ edges. Now we construct minimum 2-edge connected spanning subgraph of SR_2 using 2 copies of minimum 2-edge connected spanning subgraph of S_2. See Figure 7(a).

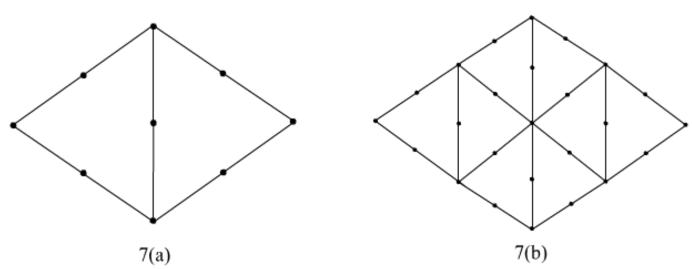

Figure 7:(a) $\varepsilon(S(SR_2) = 10$ and (b) $\varepsilon(S(SR_3) = 32$

By Lemma 2.1, no edge can be deleted from Figure 7(b). Thus $S(SR_2) = 2\varepsilon(S(S_2))$. Since 2^{2-1} edges are shared by both S_2, $\varepsilon(S(SR_2) = 2\varepsilon(S(S_2)) - 2^{2-1} = 2(2 \times 3^{2-1}) - 2^{2-1}$

We assume that the result is true for $r = k$ (i.e.) $\varepsilon(S(SR_k)) = 2\varepsilon(S(S_k)) - 2^{k-1} = 2(2 \times 3^{k-1}) - 2^{r-1}$. Consider $r = k + 1$. SR_{k+1} contains two copies of S_k. Construct a minimum 2-edge connected spanning subgraph of SR_{k+1} using two copies of minimum 2-edge connected spanning subgraph of S_{k+1} where $2^{(k+1)-1}$ edges are shared by two S_k. Thus $\varepsilon(S(S_{k+1})) = 2\varepsilon(S(S_{k+1})) - 2^{(k+1)-1} = 2(2 \times 3^{(k+1)-1}) - 2^{(k+1)-1}$. □

4 Brother Cell

Definition 4.1. *[14] Assume that k is an integer with $k \geq 2$. The kth brother cell $BC(k)$ is the five tuple $(G_k, w_k, x_k, y_k, z_k)$, where $G_k = (V, E)$ is a bipartite graph with bipartition W(white) and B(black) and contains four distinct nodes w_k, x_k, y_k and z_k. w_k is the white terminal; x_k the white root; y_k the black terminal and z_k the black root. We can recursively define $BC(k)$ as follows:*

(1) $BC(2)$ is the 5-tuple $(G_2, w_2, x_2, y_2, z_2)$ where $V(G_2) = w_2, x_2, y_2, z_2, s, t$, and
$E(G_2 = (w_2, s), (s, x_2), (x_2, y_2), (y_2, t), (t, z_2), (w_2, z_2)(s, t)$.

(2)The kth brother cell $BC(k)$ with $k \geq 3$ is composed of two disjoint copies of $(k-1)$th brother cells
$BC^1(k-1) = (G^1_{k-1}, w^1_{k-1}, x^1_{k-1}, y^1_{k-1}, z^1_{k-1})$,
$BC^2(k-1) = (G^2_{k-1}, w^2_{k-1}, x^2_{k-1}, y^2_{k-1}, z^2_{k-1})$,
a white root x_k, and a black root z_k. To be specific,
$V(G_k) = V(G^1_{k-1}) \cup V(G^2_{k-1}) \cup \{x_k, z_k\}$,
$E(G_k) = E(G^1_{k-1}) \cup E(G^2_{k-1}) \cup$
$\{(z_k, x^1_{k-1}), (z_k, x^2_{k-1}), (x_k, z^1_{k-1}), (x_k, z^2_{k-1}), (y^1_{k-1}, w^2_{k-1})\}$,
$z_k = w^1_{k-1}$, and $y_k = y^2_{k-1}$.

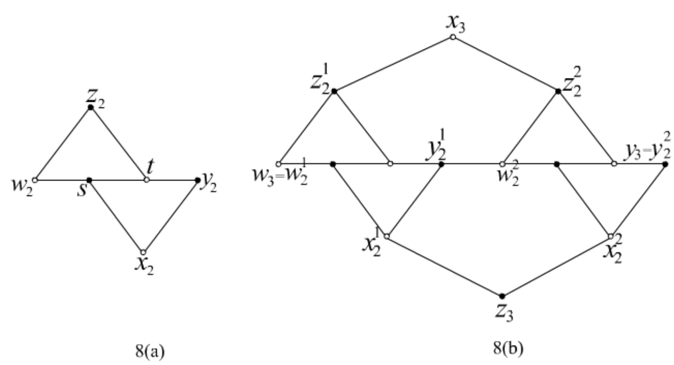

Figure 8: (a) $BC(2)$ and (b) $BC(3)$

From the definition, we construct $BC(k)$ from two disjoint copies of $(k-1)$ and each time we add five more edges $(z_k, x^1_{k-1}), (z_k, x^2_{k-1}), (x_k, z^1_{k-1}), (x_k, z^2_{k-1}), (y^1_{k-1}, w^2_{k-1})$. And each time constructing a $BC(k)$, deleting the edge (y^1_{k-1}, w^2_{k-1}) does not affect 2-edge connectivity of $BC(k)$.

Theorem 4.2. *Let $BC(r)$, $r \geq 2$ be a brother cell. Then* $\varepsilon(S(BC(r))) = 5 \times 2^{k-1} - 4$

Proof. By the Definition 4.1, BC_2 has 7 edges. Now label the vertices of $BC(2)$ as shown in the Figure 9(a). Deleting the edge(s, t), we get a cycle on 6 vertices which is a minimum 2-edge connected spanning subgraph and $\varepsilon(S(BC(2))) = 7 - 1 = 5 \times 2^{2-1} - 4 = 6$.

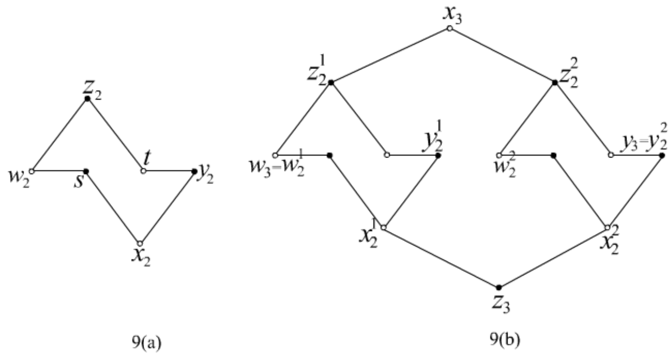

Figure 9: (a)$\varepsilon(S(BC(2))) = 6$ and (b)$\varepsilon(S(BC(3))) = 16$

We prove this theorem by induction on r. When $r = 3$, $BC(3)$ contains 2 disjoint copies of $BC(2)$ and five edges $(z_3, x^1_2), (z_3, x^2_2), (x_3, z^1_2), (x_3, z^2_2), (y^1_2, w^2_2)$ connecting theses two $BC(2)$. Now we construct minimum 2-edge connected spanning subgraph of $BC(3)$ using 2 disjoint copies of minimum 2-edge connected spanning subgraph of BC_2 and with four edges $(z_3, x^1_2), (z_3, x^2_2), (x_3, z^1_2)$, (x_3, z^2_2). See Figure 10(b). By Lemma 2.1, this is the minimum. Hence $\varepsilon(S(BC(3))) = 2\varepsilon(S(BC(2))) + 4 = 2 \times (5 \times 2^{2-1} - 4) + 4 = 16 = 5 \times 2^{3-1} - 4$.

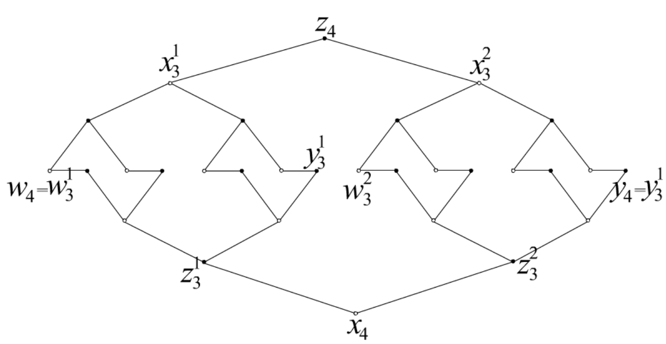

Figure 10: $\varepsilon(S(BC_4)) = 36$

We assume that the result is true for $r = k$ (i.e.) $\varepsilon(S(BC(k))) = 2\varepsilon(S(BC(k-1)))+4 = 2\times(5\times 2^{k-1}-4)+4$. Consider $r = k+1$. $BC(k+1)$ contains two copies of $BC(k)$. Construct minimum 2-edge connected spanning subgraph of $BC(k+1)$ using 2 copies of minimum 2-edge connected spanning subgraph of $BC(k)$ and with four edges $(z_k, x^1_{k-1}), (z_k, x^2_{k-1}), (x_k, z^1_{k-1}), (x_k, z^2_{k-1})$. Thus $\varepsilon(S(BC(k+1))) = 2\varepsilon(S(BC(k))) + 4 = 2 \times (5 \times 2^{k-1} - 4) + 4 = 5 \times 2^{(k+1)-1} - 4$. □

Acknowledgements

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Maximality of Circular Area: An Activity Oriented Learning

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Abstract : All we know that, by the nature of the subject, mathematical concepts are very abstract, concise and powerful in the sense that it can be applied in problems arising out of diverse field of our study. But it is observed from the direct interaction of students, their performance in examinations etc. that a significant amount of them are lacking of understanding the basic concepts of mathematics, though they do some mechanical approach towards the subject and as a consequence of which, gradually their interest towards the subject disappear. This is a great loss of a country as in the modern era, science and technology have a pivotal role and hence there is a great demand for scientifically skilled manpower and the scientific outlook is best nurtured from the logical activity i.e. in other words, mathematical activity of our brain. But it is a matter of pity that sufficient care has not been taken in the teaching-learning process of mathematics at school level. In fact research has documented that, traditional theoretical methods of teaching supplemented by activity oriented method, resulted a significant improvement of the performance level of the students. Thus development of standard books of school mathematics with adequate number of activity examples and exercise is greatly needed. In this context, in this paper,

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the author has presented different activity oriented examples concerning the circle is the shape with the greatest area than all other closed curves when the perimeter is fixed.

Keywords : secondary level, learning mathematics, mensuration, circle, solid object, circumference / perimeter, area, volume, activity.

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1 Introduction

The objective of the mathematics education at school level is to develop logical mind-set of the children. But it is a hard reality that we are far from our goal as documented from the percentage of the failure of students in different Board examinations and not to choosing science as their discipline of higher studies. The root cause behind this is their distaste in mathematics. In fact mathematics is a subject of abstract concepts built up in strictly logical framework. So teaching of this fundamental subject plays a vital role, specially, in school mathematics. The teaching methodology must have a proper blend of both theoretical and activity oriented approach so that the young mind finds a pleasure of visualizing the abstract ideas into real life situations.

In this context Psychologists, mathematicians, National Curriculum Framework[9], National Focus Group on Teaching of Mathematics of National Council of Educational Research & Training (NCERT)[10] has stressed on instruction aids and real direct experience for strengthen learning (Chakraborty, S.[2], Mondal, Dr. V. and Kar, Dr. R.[7], Roy, S.[12], Donna, H. H.[4]). Research has documented that children in early grades learn mathematics more effectively when they use physical objects in their lessons (Carmody, L.[1]; Fennema, E.[5]; Jamison, D., Suppes, P., and Wells, S.[6]; Suydam, M. and Higgins, J.[14], Rossnan, S[11], Donna H. H.[4], Selvam, Sk. P.[13]).

The use of both manipulative materials and pictorial representations is highly effective whereas symbolic treatments alone are less effective. But it is interesting to note that the great mathematician Bhaskara (1114-1183) had studied the mensuration through activity based approach in the ancient period of India (Chakraborty, S.[3]). Therefore, it is really true that there is an importance of this approach for better understanding of mathematical concepts.

At present, the status of implementation of the activities based mathematics learning in India is poor. In this regard, S. Anandalakshmy & Bala Mandir Team (2007) said in 'A Report on an Innovative Method in Tamil Nadu' on Activity Based Learning that innovative methods which engage the children and enable them to achieve mastery over school-related competencies and skills can be located here and there. However, they are small in scale and number in India.

Through in the text books of mathematics (including the book of mathematics through work) of West Bengal Board of Primary Education (WBBPE) and West Bengal Board of Secondary Education

(WBBSE) up to upper primary level some activity oriented problems have been considered but these are too in adequate. Even, in some presentations of these activities, there are apparently some ambiguities. There is no scope for alternative activities considering the target population. Even, in the text books of mathematics of class-IX & X standard of WBBSE, activity oriented learning has not been considered. On the other hand, generally, the teachers as well as the learners do not get readily available resources of activity oriented problems. So, the author as a mathematics teacher at secondary level under WBBSE feels the need to develop different activity problems in school mathematics and present them sequentially considering the learners' ability level, target group etc.

In this article, author has presented below four activities for finding, for a fixed perimeter, circle is the curve enclosing greatest area than all other closed curves which will encourage the learners to consolidate their knowledge and to practice mathematics joyfully, and surely, they will relish the simplicity & the logical beauty of the subject.

2 Objective of the Study

The aim of this study is to develop various learning activities for finding, for a fixed perimeter, circle is the curve enclosing greatest area than all other closed curves.

3 Activities

Activity-1

Verifying, for a fixed perimeter, circle is the curve enclosing greatest area than all other closed curves on a graph paper/gridlines sheet.

Requirements	: Graph sheet/ gridlines sheet, geometrical instrument box.
Mode	: Pair group.
Strategy	: Learning through activities.
Objective of the development	: Cognitive development.

Activity Follows:

Stage-I

The teacher will do the following activity with the help of the learners.

1. After showing a gridlines sheet/graph sheet, the teacher will ask the learners to choose a suitable unit of area?

2. Showing a region made of square regions, the teacher will ask the learners-how many squares are in the region?

3. Showing a region made of square and triangular halves of the squares, the teacher will ask the learners-how many squares are there (taking two triangular halves as a full square)?

Stage-II

The learners will do the following activities with the help of the teacher, if needed.

Each pair group:

1. Takes a graph sheet/ gridlines sheet.
2. Draws a closed curve like *triangle* whose perimeter is 22 cm (say) on the graph sheet/gridlines sheet. (pl. see-Figure1)
3. Identifies unit square and half of the unit square.
4. Counts the no. of unit squares and halves of the unit squares.
5. Finds the total area after counting the total unit squares and halves of the unit squares.
6. Draws a *rectangular closed curve* whose perimeter is 22 cm. (pl. see-Figure 2)
7. Identifies unit square.
8. Counts the no. of unit squares.
9. Finds the total area after counting the total unit squares.
10. Draws a *square* whose perimeter is 22 cm. (pl. see-Figure 3)
11. Identifies unit square.
12. Counts the no. of unit squares.
13. Finds total area after counting the total unit squares.
14. Draws a *pentagon* whose perimeter is 22 cm. (pl. see-Figure 4)
15. Identifies unit square and marks half of the unit square.
16. Counts the no. of unit squares and halves of the unit squares.
17. Finds total area after counting the total unit squares and halves of the unit squares.
18. Draws a *circle* whose perimeter is 22 cm. (pl. see-Figure 5) (taking radius 3.5 cm)

19. Identifies unit square and marks half of the unit square.

20. Counts the no. of unit squares and halves of the unit squares.

21. Finds total area after counting the total unit squares and halves of the unit squares.

22. Compares the areas of the calculated out the figures: triangle, square, pentagon and circle.

All pair groups:

1. Compare their results.

The work is illustrated below:

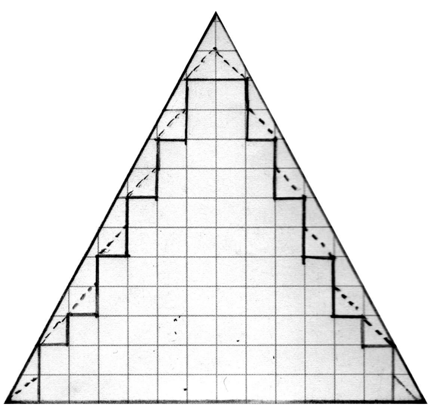

Figure 1: Triangle

No. of complete squares= 74

No. of half squares= 14

Total Area = $(74 + 14 \times \frac{1}{2})$ sq. units

= $(74 + 7)$ sq. units= 81 sq. units

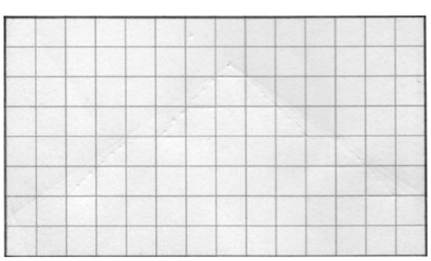

Figure 2: Rectangle

No. of complete squares= 112

Total Area = 112 sq. units

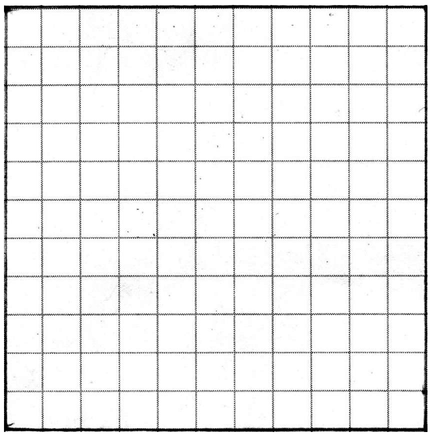

Figure 3: Square

No. of complete squares= 121

Total Area= 121 sq. units

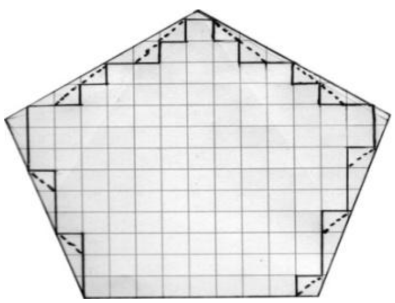

Figure 4: Pentagon

No. of complete squares= 116

No. of half squares= 13

Total Area= $(116 + 13 \times \frac{1}{2})$ sq. units = $(116 + 6.5)$ sq. units= 122.5 sq. units

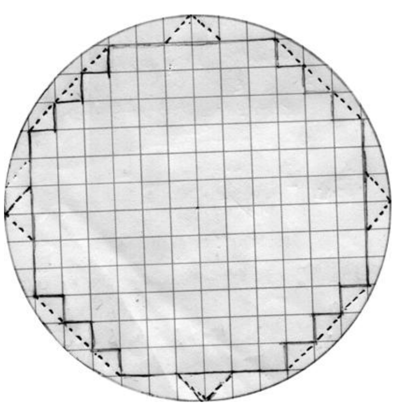

Figure 5: Circle

No. of complete squares= 120

No. of half squares= 20

Total Area= $(120 + 20 \times \frac{1}{2})$ sq. units = $(120 + 10)$ sq. units= 130 sq. units

Since 81 sq. units < 112 sq. units < 121 sq. units < 122.5 sq. units < ... <130 sq. units,

so with the same perimeter, triangular area < rectangular area < square area < pentagonal area < ... < circular area.

Activity-2

Verifying, for a fixed perimeter, circle is the curve enclosing greatest area than all other closed curves on mm division graph paper.

Requirements : mm division graph paper, geometrical instrument box.
Mode : Pair group.
Strategy : Learning through activities.
Objective of the development : Cognitive development.

Activity Follows:

Stage-I

The teacher will do the following activity with the help of the learners.

1. After showing a mm division graph paper, the teacher will ask the learners about the units of area like cm square, half cm square, mm square, half mm square.

2. Showing a region within which there are cm squares, triangular halves of cm squares, mm squares, triangular halves of mm squares, the teacher will ask the learners to find the area of the region?

Stage-II

The learners will do the following activities with the help of the teacher, if needed.

Each pair group:

1. Takes mm division graph paper sheet.

2. Draws a closed curve like *triangle* whose perimeter is 22 cm (say) on the graph sheet.

3. Identifies cm square, half cm square, mm square, half mm square.

4. Counts the no. of cm square, half cm square, mm square, half mm square.

5. Finds the total area after counting the total cm square, half cm square, mm square, half mm square.

6. Draws a *rectangular closed curve* whose perimeter is 22 cm.

7. Identifies cm square.

8. Counts the no. of cm square.

9. Finds the total area after counting the total cm square.

10. Draws a *square* whose perimeter is 22 cm.
11. Identifies cm square, half cm square.
12. Counts the no. of cm square, half cm square.
13. Finds total area after counting the total cm square, half cm square.
14. Draws a *pentagon* whose perimeter is 22 cm.
15. Identifies cm square, half cm square, mm square, half mm square.
16. Counts the no. of cm square, half cm square, mm square, half mm square.
17. Finds total area after counting the total cm square, half cm square, mm square, half mm square.
18. Draws a *circle* whose perimeter is 22 cm. (taking radius 3.5 cm)
19. Identifies cm square, half cm square, mm square, half mm square.
20. Counts the no. of cm square, half cm square, mm square, half mm square.
21. Finds total area after counting the total cm square, half cm square, mm square, half mm square.
22. Compares the calculated areas of the figures: triangle, square, pentagon and circle with a given perimeter.

All pair groups:

1. Compare their results.

Activity-3

Verifying, for a fixed perimeter, circle is the curve enclosing greatest area than all other closed curves using different shapes of cylindrical objects where the perimeters of the base of each object are same.

Requirements	: Five cylindrical objects (like measuring cylinder) whose bases are triangular, rectangular, square shaped, pentagonal and circular with equal perimeters, water, measuring cylinder, scale.
Mode	: Pair group.
Strategy	: Learning through activities.
Objective of the development	: Cognitive development.

Activity Follows:

Stage-I

The teacher will do the following activities with the help of the learners.

1. The teacher will collect five cylindrical objects of different shapes which are mentioned in the requirements.

2. After showing these cylindrical objects, the teacher will ask the learners about i) the volumes of them ii) relationship between the areas of the bases and volumes of these objects.

Stage-II

The learners will do the following activities with the help of the teacher, if needed.

Each pair group:

1. Takes the five cylindrical objects (like measuring cylinder) whose bases are triangular, rectangular, square shaped, pentagonal and circular with equal perimeter.

2. Pours these five cylindrical objects with the water of same volume (using measuring cylinder).

3. Observes the height of the water level of each cylindrical object by scale.

Activity-4

Verifying, for a fixed perimeter, circle is the curve enclosing greatest area than all other closed curves using cylindrical shaped polythene pipe.

Requirements	: Measuring Cylinder, cylindrical shaped polythene pipe, water.
Mode	: Pair group.
Strategy	: Learning through activities.
Objective of the development	: Cognitive development.

Activity Follows:

Stage-I

The teacher will do the following activities with the help of the learners.

1. After showing a measuring cylinder, the teacher will ask the students how to measure the volume of liquid contained in this cylinder.

Stage-II

The learners will do the following activities with the help of the teacher, if needed.

Each pair group:

1. Takes cylindrical shaped polythene pipe.
2. Pours about half of the cylindrical pipe with water.
3. Observes the height of the water level in the pipe.
4. Presses slowly the polythene cylinder in such a way that its base becomes non circular.
5. Observes the heights of the water levels.
6. Considers the relationship between the heights of the water levels and areas of the bases of the cylindrical pipe.

4 Conclusion

1. Four illustrations of activities for verifying that, for a fixed perimeter, circle is the curve enclosing greatest area than all other closed curves have been developed and presented sequentially in this paper. None of these activities has been appeared in the text books of WBBSE. These activities have not been stressed in the text books of NCERT. Therefore, it is a gap both in the syllabus and in the text books of school mathematics.
2. Through the illustrations of activities, it has have been shown that with a given perimeter, triangular area $<$ rectangular area $<$ square area $<$ pentagonal area $< \ldots <$ circular area.
3. This study will help the teachers and the students to show that for a fixed perimeter, circle is the curve enclosing greatest area among all other closed curves through the activity based learning.
4. Collection of multiple numbers of activities will help the teachers to choose the appropriate activity for the learners considering the learners' ability levels, needed time, availability of working materials and class room ambience etc.
5. This study will also help to (a) prepare a proper syllabus; (b) develop a good text book; (c) improve the quality of teaching-learning process of mathematics.
6. These type of activities will help the children to enjoy learning mathematics so that the phobia in mathematics will be reduced and stop the drop out of students.
7. Special interest towards mathematics can be enhanced which will be helpful for entire science education.

5 Further Study

All activities may be applied on large number of samples of class-V, VI etc for verifying that, for a fixed perimeter, circle is the curve enclosing greatest area than all other closed curves.

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